3/12/12 PRESENTATION PROPOSED ALASKA MATHEMATICS STANDARDS

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STATE OF ALASKA

SEAN PARNELL, GOVERNOR

Department of Education & Early Development

Office of the Commissioner

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March 9, 2012

Alaska House Education Committee:

The House Education Committee, through Chair Alan Dick, asked the department to be prepared to discuss with the House Education Committee the proposed Alaska mathematics standards on March 12, 2012, and the proposed language arts standards on March 16, 2012. Several specific questions were posed, as referenced in the accompanying letter. I will take advantage of this opportunity to address issues raised in this letter, and deeper discussion can occur during the upcoming hearings.

Between February 2010 and November 2011 the department held eight meetings to develop the standards. Two-hundred twenty-eight people participated in those meetings. Most of the participants were educators from diverse regions across the state, as documented in an email sent to Representative Dick's staff on November 22, 2011. Twenty-eight of the individuals who participated were from non-K-12 programs, including industry, career-technical education programs and universities. Many people from industry were invited, and approximately ten industry and career-technical training stakeholders participated.

As requested, I am enclosing in this letter as one of the attachments a document, previously provided to Representative Dick's staff, which outlines who participated in the development of these standards during the eight meetings conducted by the department. This listing provides the name, affiliation, and home city of each participant. I have not provided contact information, however, as I did not seek their permission to do so when they agreed to serve.

In December the State Board, at their quarterly meeting, reviewed the standards and voted to put them out for public comment until May 2012. During the period of public comment the department has deployed multiple strategies to seek additional comment from stakeholders, including holding webinars about the standards. The department will have conducted six webinars each on the mathematics and language arts standards prior to April 17. The department will host meetings seeking input from industry in different regions of the state, including one via audio/webinar to allow for people to participate regardless of location. In June the State Board will review the entirety of the written public comment, take public comment in person, and will consider taking action.

The next webinar regarding the language arts standards will take place March 20, and the mathematics standards webinar will be March 21. To register go to:

http://www.eed.state.ak.us/tls/assessment/2012AKStandards.html

The question was also asked who is the audience is for the standards. Academic standards are technical documents written for teachers, writers of curriculum, and writers of assessments -- those responsible for implementing the standards. The proposed standards use the technical terms of education, language arts and mathematics. Mathematics in particular, especially in the higher grades, uses complicated technical terms and concepts.

If the State Board approves the standards, the department will create parent guides for each grade containing a summary in plain English of what students should know and be able to do in English language arts and mathematics. Note that even the National PTA's parent guides use technical terms to describe higher mathematics.

I have attached a document, titled "Memo on standards to the House Education Committee." This document explains what academic standards are, what are meant by college and career ready, and provides an excellent link for additional information. I am also attaching slides from a PowerPoint that provides an overview to the standards, and Deputy Commissioner Morse will address these slides during the March 12, 2012, House Education hearing.

I also want to make the committee aware of a research study commissioned by the department, and being conducted by the Center for Alaska Educational Policy Research (CAEPR) at UAA. This is a validity study examining courses at career and technical certificate-granting institutions as well as two- and four-year degree-granting institutions. The instructors in these institutions will provide specific information about their respective courses in relation to the proposed standards in reading, writing, and mathematics, ranking the standards by importance for entering students to know. Additionally, each instructor will be able to add comments that will address standards they feel are missing. This validity study will help the State Board know, prior to taking final action, if the standards match up with the expectations necessary to enter career-technical education programs and universities in Alaska.

In the letter there are several questions about the standards documents as well as the standards themselves. I believe the webinars, which were previously addressed in this letter, will provide an excellent background in the standards; however, to really address these questions in an authentic manner it would be best to see content standards in practice. To have an academic discussion about the standards may be interesting, but to see the standards in the context of a classroom where teachers are having children address complex problems is much more instructive. We had the opportunity to see state standards implemented in both math and science when the Task Force on Theme Based Education met in Barrow. Both lessons used the Inupiaq Learning Framework as the basis for instruction. I would be happy to work with the Juneau School District to identify teachers that could provide this experience to the full committee by conducting a field visit to a local school. I am confident that we could do that in a timely manner.

The discussion regarding how the standards are assessed will provide an outline of the next step involved in building or selecting an assessment tool. Ideally the department would conduct this process following board action and be prepared to use a new assessment in 2016. To develop the test items, test blueprints and then item proposals will have to be built and vetted by Alaskans prior to field testing the items with Alaska students. This process, which engages educators and experts from the field of behavioral measurement, must be carefully crafted to ensure that a valid and reliable test is created for Alaska students.

Finally, let me address how the adopted Cultural Standards become a part of the content standards. The Cultural Standards, as well as many elements and values that are important to local communities -- such as geography, local industry, weather patterns, and community norms -- all become part of the curriculum and instruction. The standards are a statement of the content that must be learned by students; however, the curriculum provides the rich context in which those standards and many other important elements to learn are outlined. When students perform well on their local curriculum, they also will perform well on the statewide assessments as long as we build those assessments in a valid, reliable, and transparent manner.

I apologize but I will not be able to attend the hearing as I have accepted an invitation from the Chief Justice of the Alaska Supreme Court to participate in a National Leadership Summit on School Justice. In my absence Deputy Commissioner Les Morse will be present, and will be prepared to address the committee.

Sincerely,

Mike Hanley
Commissioner

Attachments:
- Letter from Chair of House Education Committee
- Memo to the House Education Committee
- Standards Development participant List
- Standards FAQ
- PowerPoint on Proposed Standards

Alaska State Legislature

Rep. Lance Pruitt, Vice-Chairman
Rep. Sharon Cissna
Rep. Eric Feige

Rep. Peggy Wilson
Rep. Paul Seaton
Rep. Scott Kawasaki

Rep. Alan Dick, Chairman
HOUSE EDUCATION COMMITTEE

March 7, 2012

Commissioner Mike Hanley
PO Box 110500
Juneau, AK 99811

Re: **New Math Standards**

Dear Commissioner Hanley:

On Monday March 12, the House Education Committee will begin hearings on the proposed Alaska State Educational Standards. We will begin with the Math Standards, and go through them with appropriate detail.

Please be prepared to comment on and answer questions regarding the following:

How many public hearings for the proposed State Educational Standards have already occurred?

Who was the target audience for the proposed standards? Teachers, curriculum coordinators, parents?

Please give the names and contact numbers for those who 1) participated in the development of the proposed Alaska State Educational Standards **and** 2) were ***not*** educators.

In the introduction to the Mathematics standards (p.3), is the statement that "They (the standards) simply establish a strong foundation of knowledge and skills all students need *for success* after graduation." As we discuss these standards, please be prepared to show how each of these skills will prepare students "for success," ie. real-life problems.

Please be prepared to explain and give examples for the *Standards for Mathematical Practice* on pages 4-15 in the proposed Alaska Mathematics Standards.

In the *Description of Mathematics Standards* (p. 3) it states: The high school standards set a rigorous definition of readiness by helping students develop a depth of understanding and ability to apply mathematics to novel situations, as college students and employees *regularly* do.

Please give a few examples of applying mathematics to novel situations.

On Page 4 of the same section of the Mathematics Standards, is the statement: These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as *sensible, useful, and worthwhile,* coupled with a belief in diligence and one's own efficacy).

Please explain the above and give examples of "productive disposition" that will be promoted by these standards.

Also on Page 4 of the Mathematics Standards:
What is meant by the following? How is a teacher to apply them?
3. Construct viable arguments and critique the reasoning of others
5. Use appropriate tools strategically
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

From Pages 4-5 of the Mathematics Standards for the following, please give examples. Pick any real life situation.

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?"

Please describe examples of the following. **In grades 3-5 mathematically proficient students will**:

- explain correspondences between equations, verbal descriptions, tables, and graphs
- draw diagrams of important features and relationships, graph data, and search for regularity or trends
- understand the approaches of others to solving complex problems
- identify correspondences between different approaches
- check if the solution makes sense

Please describe examples of the following. I can see teachers doing these, but struggle seeing all students doing them. How would a teacher approach teaching these strategies?

In grades 6-8 mathematically proficient students will:

- explain correspondences between a new problem and previous problems
- represent algebraic expressions numerically, graphically, concretely/with manipulatives, verbally/written
- explain connections between the multiple representations
- determine the question that needs to be answered
- make a plan for attempting a problem
- choose a reasonable strategy
- identify the knowns and unknowns in a problem
- solve a problem in more than one way

Please give examples of the following using real life problems. How would a teacher approach teaching these strategies?

In grades 9-12 mathematically proficient students will:

- make connections between a new problem and previous problems
- determine the question that needs to be answered
- make a plan for attempting a problem
- choose a reasonable strategy
- identify the knowns and unknowns in a problem
- break a problem into manageable parts or simpler problems

- represent algebraic expressions numerically, graphically, concretely/with manipulatives, verbally/written
- explain connections between the multiple representations
- solve a problem in more than one way
- explain the meaning of a problem and look for an entry point
- analyze a problem and make a plan for solving it
- explain correspondence between differing approaches to identify regularity and trends
- check answer using a different method
- identify correspondence between different approaches
- monitor and evaluate progress and change course if necessary
- check the answers to problems using a different method and continually ask, "Does this make sense?"

For the above, how would DEED assess the student's performance on the Standards Based Assessment and the High School Graduation Qualification Exam?

As the standards call for districts to implement the Cultural Standards, please give three examples of how they might do that using the high school standards. Use examples from any of the five Alaska Native cultural areas.

I look forward to our discussion.

Sincerely,

Representative Alan Dick Chairman

MEMORANDUM

State of Alaska
Department of Education
& Early Development

To: House Education Committee

Date: March 8, 2012

Phone: 465-2802

Fax: 465-4156

From: Mike Hanley
Commissioner

Subject: State standards

What are academic standards?

Academic standards are technical documents written for teachers, writers of curriculum, and writers of assessments. The proposed standards use the technical terms of education, language arts and mathematics. Mathematics in particular, especially in the higher grades, uses complicated technical terms and concepts.

If the State Board approves the standards, the department will create parent guides for each grade containing a summary in plain English of what students should know and be able to do in English language arts and mathematics. Note that even the National PTA's parent guides use technical terms to describe higher mathematics.

The proposed state standards answer the question of what we should strive to teach in each grade, with one grade building on previous grades and leading to the next grades, in English language arts and mathematics. For example, the standards tell us what fifth-graders should be taught in mathematics. That is a technical question. To answer it, you need to know mathematics, know how to teach mathematics, and understand the development of children.

To answer that sort of question, you also need to know what you're aiming for in English language arts and mathematics skills in a high school graduate. We do know what the end-users of our graduates want. More than half of our graduates need remedial courses in English and/or mathematics at the University of Alaska. Nationwide, a quarter of high school graduates who apply to the military cannot pass its entrance exam, the Armed Services Vocational Aptitude Battery. Across the country, business groups have expressed dissatisfaction with high school graduates.

Alaska's proposed standards are detailed. It takes 13 years for students to work their way through the standards and complete their public education. It takes skilled teachers a full school year to teach a year's worth of standards by translating these complicated ideas into terms that students can understand. That is the art and craft of being a teacher.

State standards do not introduce complexity and change in education. Those qualities have always existed. If we did not have state standards and had only district-level curricula, teachers would still have the task of translating complicated ideas into terms that students can understand. Districts would still have to indoctrinate newly hired teachers into their curricula, and districts would still revise their curricula every few years -- just as they did before state standards existed.

Complexity and change are part of private enterprise, as well. Some businesses practice a model of continuous improvement.

<u>What do states mean by the terms "college ready" and "career ready"?</u>

When we say "college," we mean every type of schooling and training after high school: the military, union apprenticeships, vocational certificate programs, associate's degrees in careers, and bachelor's degrees in careers and academic fields. Specifically, when we say "college-ready," we mean that high school graduates should not need remedial courses in their postsecondary institutions.

Many occupations require some sort of training beyond high school. Thus, the term "college-ready" applies to the needs of many students, far more than the few Alaskans who complete a bachelor's degree. There is an overlap in required skills for postsecondary technical training and academic degrees.

For example, the construction technology associate's degree from the University of Alaska Southeast requires students to take an algebra course that compares to an intermediate high school algebra course. If you ask that student when he's 14 years old whether he wants to study algebra, he might say no. He would not know whether algebra would be relevant to his adulthood.

What does "career ready" mean? There are hundreds of careers, and obviously students don't graduate high school ready to be electricians and doctors.

National associations that promote career and technical education list three criteria of career readiness: 1) academic preparation in high school; 2) personal traits and skills that employers value, such as being on time, behaving and dressing appropriately, being able to problem-solve and collaborate with others; and 3) training for a specific career.

The proposed standards in English language arts and mathematics meet the first criteria for those two fundamental subjects. The standards are not intended to deal with the second criteria, but they do not preclude schools from dealing with workplace readiness in other ways. The standards prepare students to be able to enter postsecondary training and meet the third criteria.

It is important to note what the proposed standards are and what they are not. Standards in English language arts and mathematics do not preclude having other sets of standards such as for STEM, culture, career and technical education, entrepreneurship, or workplace readiness. In fact, the English language arts and mathematics standards could be partly taught through those subjects. For example, mathematics is a good preparation for engineering; English and mathematics have value for entrepreneurs.

Why have academic standards?

The goal of formal education is not to meet students where they are and leave them there. If only 7% of our ninth-graders receive a bachelor's degree within six years of leaving high school, that is a cause for alarm. It is not a target to aim for. Nor should it be interpreted as a reflection of student desires. Rather, it reflects the limits that all of us responsible for public education have placed on those students.

An indication of student desires is the retention rates at the University of Alaska. Only about a quarter of freshmen seeking a B.A. at UA graduate within six years. Only about a third of freshmen seeking a technical certificate, associate's degree or bachelor's degree graduate within six years. In other words, three times as many Alaskans try for a UA certificate or degree as actually receive one. And we don't know how many other students wanted a credential but never entered postsecondary education.

Surely, there are many causes of this. We believe that some of the students' limitations are academic. Our evidence is that UA itself requires at least half of its students to take remedial (meaning high school-level) courses in English and mathematics. For example, in fall of 2009 UA had 3,587 first-time freshmen, of which 1,902 took remedial courses -- about 53%.

The proposed standards in English language arts and mathematics set high expectations for students and the public education system. The mathematics standards distinguish between standards that all students should strive for and standards for students taking advanced courses. The goal is to give students choices in life after high school.

We did not propose the English language arts and mathematics standards because we think they are the only important subjects in school, or because we think that school is the only source of valued knowledge.

We proposed the English language arts and mathematics standards because:

1) those subjects are fundamental to other school subjects and to training after high school on the job, in a technical school, the military, or college; and

2) the study of those subjects develops students' minds and habits of thought.

Students who study English language arts and mathematics learn to practice attentiveness, attention to detail, perseverance, creativity, problem-solving, individual and group learning, and complex and nuanced thinking.

One of the flaws of educating children for what seems relevant now is that as adults they might need to be very flexible in moving from job to job as the economy shifts. We must educate children for an adulthood that they and we cannot fully anticipate. The study of English language arts and mathematics is part of such a foundation.

State standards are an important element in equitable public education. Some of the strongest supporters of No Child Left Behind have been civil rights and disabilities organizations because they want schools to act on the premise that all children can learn and all will be held to the same standards.

We cannot agree with the characterization of standards as a fad. Every industrialized nation has them. Forty-five states and D.C. have adopted the Common Core standards, which generally are more rigorous than the states' original standards were. Over time, educational materials will be geared to the Common Core and similar standards. College and career-ready standards may be required under a reauthorized federal Elementary and Secondary Education Act. They are certainly required for a waiver from No Child Left Behind. They have been required for certain federal education funds.

<u>Standards alone are ineffective</u>

Admittedly, standards alone do not increase student achievement. School districts' curricula, professional development and teachers' instruction must incorporate the standards, or there will be no effect on the students.

Randi Weingarten, president of the American Federation of Teachers, recently published a statement called "Implementation Will Move the Needle." She said the Common Core standards (which are similar to Alaska's proposed standards) "are an essential building block for a better education system – not a new educational craze, a federal intrusion, or an untested idea." Weingarten said the standards must be supported by development of aligned curriculum, professional development, instructional materials, collaborative planning, and aligned assessments that inform instruction but are not used excessively or punitively. We agree.

Massachusetts set high standards more than a decade ago and worked through the challenges of implementing them. The result today is that Massachusetts is the highest-scoring state on the National Assessment of Educational Progress in its overall student body and in every subgroup of students. In other words, the percentage of African-American students in Massachusetts who score proficient on the NAEP is the highest of the states and D.C. The same is true of their Hispanic students, students with disabilities, low-income students, and English language learners. Massachusetts is the only American state whose students score comparably on international assessments with leading nations such as Finland, Japan, Taiwan and South Korea.

It is important not to overpromise the effect of standards. Massachusetts has raised all boats, but it still has achievement gaps. Some of its students still need remedial courses in college. In fact, many of its students still don't score proficient on assessments (which is equally true of high-scoring nations). But Massachusetts has seen progress in proficiency rates, and it is giving parents and students an accurate understanding of whether a student is proficient.

Final thoughts

The costs of implementing more rigorous standards must be weighed by the costs of standing still. The costs of standing still will be borne by families. Every time an Alaska high school graduate spends money to attend UA and leaves without a diploma, his or her family has lost their investment. Every Alaskan high school graduate who is undereducated and has not reached his or her potential faces a greater risk of reduced earnings, unemployment, and reliance on public support.

For more information on Alaska's proposed standards, see
http://www.eed.state.ak.us/standfaqs.html

For comments on the Brookings Institution report, which criticized the effectiveness of standards, see
http://education.nationaljournal.com/2012/02/common-cores-good-bad-and-ugly.php

First Name	Last Name	School District/Campus Name	School	City
Kathy	Ahgeak	North Slope Borough School District		Barrow
J. Paul	Apfelbeck	Galena City Schools		Galena
Anne	Armstrong	University of Alaska Fairbanks		Fairbanks
Christina	Barlow	Craig City Schools	Craig High School	Craig
Phyllis	Barnett	Saint Mary's Schools		St. Mary's
Anne	Barnett	Anchorage Schools		Anchorage
Susan	Baxter	Alaska's Environmental Literacy Plan		Juneau
Sandra	Bell	Fairbanks North Star Borough Schools	Anderson Elementary School	Eielson AFB
Thomas	Belleque	Bering Strait Schools		Teller
Ray	Benson	Lower Kuskokwim Schools		Atmautluak
Matthew	Bierer	Fairbanks North Star Borough Schools		Fairbanks
Rebecca	Binkley	Southeast Island Schools		Thorne Bay
William	Black	Nenana City Schools	District Office	Nenana
Brian	Blitz	University of Alaska Southeast		Juneau
Mary	Bogard	Delta-Greely Schools		Delta Junction
Carole	Bookless	Juneau Borough Schools		Juneau
Gerry	Briscoe	Southeast Alaska Regional Resource Center		Anchorage
Valerie	Brooks	Ketchikan Gateway Borough Schools	Houghtaling Elementary	Ketchikan
Peggy	Bruno	Yukon-Koyukuk Schools		Huslia
Jeanne	Campbell	Kashunamiut Schools		Chevak
Arva	Carlson	Nine Star		Anchorage
Douglas	Carroll	Cordova City Schools		Cordova
Jamie	Carroll	Fairbanks North Star Borough Schools		Fairbanks
Nina	Chordas	University of Alaska Southeast		Juneau
Kathy	Christopherson	Lower Kuskokwim Schools		Bethel
Russell	Clark	Lower Yukon Schools	Pilot Station School	Pilot Station
Deanna	Claus	Craig City Schools		Craig
Carol	Combs	Anchorage Schools		Anchorage
Gary	Cooper			Anchorage
Sandra	Cott	Lower Kuskokwim Schools		Bethel
Cathy	Coulter	University of Alaska Anchorage		Anchorage
Barb	Crandall	Anchorage Schools		Anchorage
Samuel	Crow	Lower Kuskokwim Schools		Bethel
Michelle	Daml	Fairbanks North Star Borough Schools		Fairbanks
Jodi	Darling	Bering Strait Schools		Stebbins
Rodger	Davis	Take Flight Alaska		Anchorage

First Name	Last Name	School District/Campus Name	School	City
John	DeVolld	Kenai Peninsula Borough Schools		Soldotna
Deborah	Dixon	AVTEC - Alaska's Institute of Technology		Seward
Benjamin	Dolgner	Bering Strait Schools		Stebbins
Jean	Domansi	Lower Yukon Schools		Mountain Village
Teresa	Duncan	Dillingham City Schools		Dillingham
Hugh	Dyment	Lower Kuskokwim Schools		Bethel
Sarah	Edwards	Dillingham City Schools		Dillingham
Shelly	Eldred	Anchorage Schools		Anchorage
Mary	Ellison	Anchorage Schools		Anchorage
Deborah	Endicott	Southwest Region Schools		Dillingham
Sara	Erickson	Lake and Peninsula Borough Schools		
Ben	Eveland	Career and Technical High School		Wasilla
Philip	Farson	Anchorage Schools		Anchorage
Michael	Fenster	Anchorage Schools		Anchorage
Amanda	Ferrari	Pribilof Schools		St. Paul
Dora Judith	Ferri			Fairbanks
Robert	Fish	Kodiak Island Borough Schools	Kodiak High School	Kodiak
Terry	Folsom	Valdez City Schools		Valdez
Victoria	Foote	University of Alaska Fairbanks - Rural Health Prog		Fairbanks
Emily	Forstner	Mat-Su Borough Schools		Wasilla
Michelle	Foss	Anchorage Schools		Anchorage
Virgil	Fredenberg	UAS School of Education		Juneau
Ron	Fuhrer	Anchorage School District		Anchorage
Marina	Gantz	Anchorage Schools	Boniface Educational Center	Anchorage
Pamela	Garcia	Juneau Borough Schools		Juneau
Monica	Garza	Yukon-Koyukuk Schools		Minto
Rebecca	Gerik	Anchorage Schools		Anchorage
Dianna	Gharst	Bering Strait Schools		Elim
Matthew	Gho	Anchorage Schools		Anchorage
Julia	Gibeault	Anchorage Schools	North Star Elementary	Anchorage
Kim	Girard	Anchorage Schools		Anchorage
Amber	Glynn	Delta/Greely Schools	Delta Junction Sr. High School	Delta Junction
Diana	Grady	Bering Strait Schools		Unalakleet
Cindy	Granatir	North Slope Borough Schools		Barrow
Kirsten	Gray	Fairbanks North Star Borough Schools		Fairbanks
Jessica	Graziano	Anchorage Schools		Anchorage

First Name	Last Name	School District/Campus Name	School	City
David	Grimes	Kake City Schools	Kake Elementary & High School	Kake
Alison	Gryga	Kashunamiut Schools		Chevak
Paul	Gutzler	Kenai Peninsula Borough Schools		Homer
Cheryl	Guyett	Anchorage A.J. Dimond High School		Anchorage
Joseph	Hackenmueller	Anchorage School District		Anchorage
Teresa	Hall	Fairbanks North Star Borough Schools		Fairbanks
John	Hamill	Alaska Military Youth Academy		Fort Richards
Kimberly	Handy	Anchorage Schools	East High School	Anchorage
Erik	Hanson	Kodiak Island Borough Schools		Kodiak
Susan	Hardin	Petersburg City Schools		Petersburg
Ranada	Hassemer	Kenai Peninsula Borough Schools		Soldotna
Nicole	Havert	Mat-Su Borough Schools		Palmer
Jeanette	Hayden	Fairbanks Hutchison High School		Fairbanks
Krista	Heard	Department of Education		Juneau
Dona	Helmer	Anchorage Schools		Anchorage
Martina	Henke	Anchorage Schools		Anchorage
Rebecca	Himschoot	Sitka Borough Schools		Sitka
Todd	Hindman	Nome City Schools	Anvil City Science Academy	Nome
Kimberly	Hoover	Southeast Island Schools		Naukati
Susan	Hubbard	Kuspuk Schools		Sleetmute
Matthew	Hunter	Mount Edgecumbe		Sitka
Ann	Ibele	Anchorage Schools		Anchorage
Marcia	Indahl	Anchorage Schools		Anchorage
Karen	Iris	Anchorage Schools		Anchorage
Amy	Iutzi	Department of Labor		Juneau
Matt	Johnson	Bering Strait Schools		Stebbins
Kimberly	Johnson	Kenai Peninsula Borough Schools		Anchor Point
David	Jones	Ketchikan Gateway Borough Schools		Ketchikan
Carolyn	Jordan	Fairbanks North Star Borough Schools	Tanana Middle School	Fairbanks
Barbara	Jordan	Sitka Borough Schools	Blatchley Middle School	Sitka
Jamie	Katchatag	Bering Strait Schools		Unalakleet
John	Keller	University of Alaska Fairbanks		Fairbanks
Amy	Kesten	Juneau Borough Schools		Juneau
Ruth	Knight	Valdez City Schools	Valdez City Schools	Valdez
David	Kohler	Anchorage Schools		Anchorage
Sally	Kookesh	Chatham Schools		Chatham

First Name	Last Name	School District/Campus Name	School	City
Joe	Krause	Nenana City Schools		Nenana
Harvey	Kurzbard	Fairbanks North Star Borough Schools		Fairbanks
Nathan	Laabs	Denali Borough Schools		Healy
Kathy	Leary	Ilisagvik College		Barrow
Lisa	Leeper	Nome Public Schools		Nome
Kim	Liland	Anchorage Schools		Anchorage
Celeste	Long	Anchorage Schools		Anchorage
Craig	Luchsinger	Kuspuk Schools	Aniak Jr./Sr. High School	Aniak
Janice	Lund	Craig City Schools	Craig Elementary	Craig
Christy	Lyle	Kodiak Island Borough Schools	Kodiak Middle School	Kodiak
Diane	Maples	Alaska Tech Prep Consortium		Anchorage
Edward	Marman	Mat-Su Borough Schools		Palmer
Mark	Martin	Denali Borough Schools		Healy
Jeni	Mason	Denali Borough Schools		Cantwell
Carol	May	Juneau Borough Schools	Thunder Mountain High School	Juneau
Romee	McAdams	Tribal Recruitment Coordinator		
Mary	McCaffrey	Skagway City Schools		Skagway
Les	McCormick	Chatham Schools		Angoon
Patricia	McDonald	EED SSOS		Juneau
Susan	McIntosh	Fairbanks North Star Borough Schools		Fairbanks
Scott	McKay	Valdez City Schools	Hermon Hutchens Elementary	Valdez
Lesa	Meath	Fairbanks North Star Borough Schools		Fairbanks
Helen	Mehrkens	Department of Education		Juneau
Debbie	Merle	Ketchikan Gateway Borough Schools		Ketchikan
Jim	Merriner	SBOE		
Vivian	Meyer	Skagway City Schools	Skagway City School	Skagway
Suzie	Michaud	Craig City Schools		Craig
Jenn	Miller	Wrangell City Schools		Wrangell
Stacy	Miller	Anchorage Schools		Anchorage
Sheila	Miller	Ketchikan Gateway Borough Schools		Ketchikan
Carolyn	Mork	Sitka Borough Schools	Keet Gooshi Heen Elementary	Sitka
Karla	Moxley	North Slope Borough Schools		Barrow
Suzan	Mullane	Anchorage Schools		Anchorage
Gretchen	Murphy			Fairbanks
Mary	Murphy	Anchorage Schools		Anchorage
Tyler	Nalisnick	Take Flight Alaska		Anchorage

First Name	Last Name	School District/Campus Name	School	City
Debbie	Narang	University of Alaska Anchorage		Anchorage
Allyson	Nicholson	Fairbanks North Star Borough Schools		Fairbanks
Doug	Noon	Fairbanks North Star Borough Schools		Fairbanks
Judy	Norton-Eledge			Anchorage
Carl	Oberg	Anchorage School District		Anchorage
Mark	O'Brien	Ketchikan Gateway Borough Schools		Ketchikan
Greg	Owens	University of Alaska Fairbanks		Fairbanks
Tammy	Palmer	Mat-Su Borough Schools		Palmer
Tina	Pasteris	Juneau Borough Schools	District Office	Juneau
Star	Patterson	Fairbanks North Star Borough Schools	Two Rivers School	Fairbanks
Philip	Patterson	University of Alaska Fairbanks		Fairbanks
Paula	Pawlowski	Alaska PTA - Parent Engagement		Anchorage
Marjorie	Payton-Hewlett	Southwest Alaska Vocational & Education Center		King Salmon
Jodi	Picou	Mat-Su Borough Schools		Palmer
Angela	Pirtle	Sitka Borough Schools	Keet Gooshi Heen Elementary	Sitka
Kathy	Port	Fairbanks North Star Borough Schools		Fairbanks
Steve	Potter	Juneau Borough Schools	Juneau-Douglas High School	Juneau
Gale	Pratt	Yukon Flats Schools		Fort Yukon
Lolly	Rader	Anchorage School District		Anchorage
Jennifer	Randall	Fairbanks North Star Borough Schools		Fairbanks
LaDonna	Rees	Anchorage Schools		Anchorage
Melissa	Reese	Valdez City Schools		Valdez
Cliff	Reimers	Anchorage Schools		Anchorage
Cathy	Rexford	North Slope Borough School District		Barrow
Mary	Richards	Anchorage Schools	South Anchorage High School	Anchorage
Anthony	Rickard	University of Alaska Fairbanks		Fairbanks
Melissa	Rickey	University of Alaska Fairbanks		Fairbanks
Amber	Rinella	Mat-Su Borough Schools		Palmer
William	Rodawalt	Dillingham City Schools		Dillingham
Roy	Roehl	University of Alaska Fairbanks		Fairbanks
Lesa	Rohrer	Northwest Arctic Borough Schools		Kotzebue
Lori	Rucksdashel	Anchorage Schools		Anchorage
Margaret	Salisbury	Fairbanks North Star Borough Schools		Fairbanks
Jessica	Schauffler	Fairbanks North Star Borough Schools		Fairbanks
Laurie	Schoenberger			Juneau
Sandy	Schoff			Anchorage

First Name	Last Name	School District/Campus Name	School	City
Deanna	Schultz	University of Alaska Anchorage		Anchorage
Jeff	Selvey	Department of Labor & Workforce Development		Anchorage
Barb	Shogren	Mat-Su Career & Tech Education Department		Palmer
Maria	Skala	Anchorage Schools		Anchorage
Sheri	Skelton	Bering Strait Schools	White Mountain School	White Mountain
Jan	Slattery	Anchorage Schools	East High School	Anchorage
Bev	Smith	Bev Smith Educational Consulting		Douglas
Alan	Sorum	Prince William Sound Community College		Valdez
Katy	Spangler	UAS School of Education		Eagle River
Sally	Spieker	University of Alaska Anchorage		Anchorage
Jamie	Stacks	Pribilof Schools	St. Paul School	St. Paul
Sarah	Stanley	University of Alaska Fairbanks		Fairbanks
Robbin	Stockton	North Slope Borough Schools		Barrow
Jennifer	Stone	University of Alaska Anchorage		Anchorage
Janice	Summers	Anchorage Schools		Anchorage
Amy	Summers	Fairbanks North Star Borough Schools		Fairbanks
Sabrina	Sutton	Kodiak Island Borough Schools	East Elementary	Kodiak
Patrick	Tatera	Galena Schools	Interior Distance Education of AK (IDEA)	Fairbanks
Dana	Thomas	University of Alaska Fairbanks		Fairbanks
Joel	Thomas	Lower Kuskokwim Schools		Bethel
Laron	Thomas	Kashunamiut Schools		Chevak
Nancy Georgia	Tompkins	Mat-Su Borough Schools	Academy Charter School	Palmer
Jo-Ann	Trozzo	Skagway City Schools		Skagway
Tamara	Van Wyhe	Copper River Schools	Kenny Lake School	Copper Center
Emily	Vanderpool	Kuspuk Schools		Aniak
Theresa	Vick	Fairbanks North Star Borough Schools		Fairbanks
John	Wahl	Juneau Borough Schools	District Office (JSD)	Juneau
Allison	Wall	Mat-Su Borough Schools		
Douglas	Walrath	Northwestern Alaska Career and Technical Center		Nome
Jack	Walsh	Bristol Bay Borough Schools		Naknek
Matt	Walton	Kenai Peninsula Borough Schools	Soldotna High School	Soldotna
Bill	Watkins	Kodiak Island Borough School District		Kodiak
Lisa	Weight	Anchorage Schools		Anchorage
Tracie	Weisz	Alaska Gateway Schools		
Seth Marie	Westfall	Anchorage Schools		Anchorage
Harry	White	Yukon-Koyukuk School District		

First Name	Last Name	School District/Campus Name	School	City
Sandra	Wildfeuer	University of Alaska Fairbanks		Fairbanks
Penny	Williams	Anchorage Schools	Boniface Educational Center	Anchorage
Lynn	Williams	EED		
Susan	Wilson	Anchorage Schools		Anchorage
Laura	Wren	Anchorage Schools		Anchorage
Samantha	Wuttig	Fairbanks North Star Borough Schools	District Office	Fairbanks
Bei	Yang	Anchorage Schools		Anchorage
Joe	Zawodny	Anchorage Schools		Anchorage
Elizabeth	Zeuli	Anchorage Schools	Rogers Park Elementary	Anchorage
Leslie	Zibell	Northwest Arctic Borough Schools		
Victor	Zinger	University of Alaska Fairbanks		Fairbanks
Holly	Zwink	Kenai Peninsula Borough Schools		

Alaska proposes new standards for English language arts and mathematics in the public schools

Speak out about Alaska's student standards

The State Board of Education & Early Development welcomes your comments on proposed new standards in English language arts and mathematics for public school students.

All public comments will go to the State Board for their consideration. The State Board, which has the authority to adopt state standards, is scheduled to decide the issue in June 2012. The State Board could choose to adopt the standards as proposed, adopt them with changes, or not adopt them.

If the new standards are adopted in June 2012, the state will work with school districts to align their curriculum to the new standards. The department expects that students would first be assessed in the new standards in spring 2016.

Why are new standards being proposed?

The new standards would guide the education of students from kindergarten through grade 12 so that Alaska's high school graduates are ready for postsecondary education, career training and the workplace. Alaska's current standards do not reach this goal for two main reasons: they stop at grade 10, and they are not rigorous enough.

Currently, many Alaska's high school graduates who enter bachelor degree programs at the University of Alaska need remedial courses in English and/or mathematics. Those students are more likely to drop out of college. Employers also say high school graduates are not fully prepared for the workplace. Only about a third of Alaska students score proficient or above on the National Assessment of Educational Progress, a rigorous test in English and math given to large samples of fourth-graders and eighth-graders in each state.

Alaska's high school graduates must be prepared to compete for jobs, even within Alaska, with people from across the United States and the globe. The proposed standards would increase our students' skills in academic English and mathematics. These skills are important in business, industry, government, and science and technology.

Briefly, how do the proposed standards differ from current standards?

The proposed standards for English language arts cover reading, writing, vocabulary, and speaking and listening. Our current grade-level standards cover only reading and writing. Speaking and listening are important skills in postsecondary classrooms and the workplace. In addition, the proposed standards expect students to develop their language skills in the context of many subjects, not just literature. This is important because employers say that employees are not always skilled in reading and writing informational documents.

Finally, as an appendix to the standards, Alaska educators will provide lists of recommended literary and informational texts that challenge students to build knowledge, gain insights, and broaden their perspective.

In mathematics, students will be taught the content of some standards a grade earlier than they are now so that students in the earlier grades are well-prepared for high school. In high school, students will develop a depth of understanding and the ability to apply mathematics to new situations, as college students and employees must do. The proposed standards align with expectations for students from the National Council of Teachers of Mathematics and the National Research Council of the National Academy of Sciences and the National Academy of Engineering.

How were the proposed new standards created?

For nearly two years, the Alaska Department of Education & Early Development has worked on the standards with rural and urban Alaskans from around the state, including representatives of universities, career and technical programs, and industries. Working groups also included teachers and other specialists in English and mathematics; school districts' curriculum coordinators; librarians; high school teachers of career and technical subjects; and teachers who work with struggling schools, students with disabilities, English language learners, economically disadvantaged students, and students of diverse ethnicities.

The Alaska reviewers compared Alaska's current standards with new nationwide standards in reading, writing and math for each grade from kindergarten to 12, and with new nationwide college-ready and career-ready standards, which define what students must know and be able to do to be ready for postsecondary education or careers.

The review process incorporated the best of Alaska's current standards, added new standards, and revised standards for clarity. Reviewers paid attention to what was developmentally appropriate to each grade level. They made sure that each grade level gradually built toward a high level of skill and understanding in grade 12 students.

The nonprofit National Center for the Improvement of Educational Assessment reviewed Alaska's proposed standards in detail. The University of Alaska will review the standards to determine whether they will lead to high school graduates who are ready for college without remediation.

The result is a set of standards created by Alaskans but comparable in rigor to new standards that are being adopted around the United States. The department expects that many curricular materials that will be developed for the new nationwide standards would be relevant to Alaska's proposed standards.

How would the new standards be used?

The proposed new standards provide a consistent, clear understanding of what students are expected to learn so that teachers and parents know what they need to do to help students.

School districts would use the standards as the framework for their curriculum in English language arts and mathematics. In turn, teachers would use their district's curriculum to determine their lesson plans.

The standards do not tell teachers how to teach. The standards do not preclude the use of culturally relevant teaching methods. The standards represent a goal. Educators locally will determine how to meet this goal.

The standards in English language arts and mathematics are not meant to imply that those are the only important subjects in school, or that school-based knowledge is the only important type of knowledge. The department proposed new standards in English language arts and mathematics because those subjects are fundamental to many other subjects in school and to many occupations and aspects of daily life.

The department would use the standards to develop assessments for students. The results of these assessments will show whether students have advanced skills, proficient skills, or less than proficient skills in English language arts and mathematics.

For families, the assessments help you understand your students' readiness for postsecondary education and careers. Classroom grades and the observations of teachers and families also show how students are performing in school.

For all Alaskans, the assessments allow us to see how students are doing schoolwide, districtwide and statewide. Citizens, policymakers and administrators locally and at the state level will be able to see the bigger picture of student achievement. At the same time, educators can look at results for subgroups of students, such as economically disadvantaged students or students with disabilities. That helps school districts understand where to target their resources.

The assessments allow Alaskans to see whether more students are becoming proficient over time. The federal government currently requires states to have standards and assessments, and to use the assessment results to hold school districts accountable for increasing student achievement. Alaska has a similar statute.

If the State Board approves the proposed new standards, what are the next steps?

The Alaska Department of Education & Early Development will prepare brochures that summarize the standards at each grade level for families and for educators. Parents, students and teachers will be able to understand the expectations for students in English language arts and mathematic at each grade.

Just as Alaskans have high expectations for students, we must have high expectations for the educational system at the local and state level. Implementing new standards requires a transition period. Students will be educated for nearly four years under the new standards before they are assessed in them.

This provides time for school districts to closely align what is taught in the classroom to the new standards. The state, as well, must align new assessments with the standards.

The department will provide school districts with a clear understanding of the differences between Alaska's former standards and the new standards, and offer online guidance in making the transition. Additionally, the department will continue to provide districts twice-yearly institutes on curriculum and alignment.

How can I comment on proposed new standards?

You can review the proposed new standards online at http://www.eed.state.ak.us/regs/, where you also can submit comments electronically.

If you would like a copy of the standards mailed to you, contact Eric Fry at 907-465-2851 or eric.fry@alaska.gov or Dorothy Knuth at 907-465-2802 or Dorothy.knuth@alaska.gov.

You may submit comments in writing by fax or mail. Fax comments to 907-465-4156 attention Dorothy Knuth. Mail comments to Alaska Department of Education & Early Development, Attn: Dorothy Knuth Regulation Comments, 801 West 10th Street, Suite 200, P.O. Box 10500, Juneau, AK 99811-0500.

The deadline to submit comments prior to the State Board meeting is May 12. If you would like to make your comments in person or telephonically at the June 7-8, 2012, State Board meeting in Anchorage, see http://www.eed.state.ak.us/State_Board/ in late May, when the agenda will be posted.

How can I read more about it?

The current standards and the proposed standards are at http://www.eed.state.ak.us/tls/assessment/2012AKStandards.html

Summary of proposed English language arts standards

The reading standards require students to read increasingly complex texts so that by the end of high school they are ready for the demands of college-level and career-level reading. The standards require that students grow in their reading comprehension; as they advance through the grades, they are able to gain more from whatever they read. The reading standards assume that students must be able to read well in all their courses. Students are expected to read challenging informational texts in subjects such as social studies and science, as well as literature.

The cornerstone of the writing standards is the ability to write logical arguments based on substantive claims, sound reasoning, and relevant evidence. The standards begin to teach these skills in the earliest grades. Students will be expected to conduct research in short, focused projects and longer in-depth projects. Research skills – in order to gather information before writing or speaking -- are emphasized throughout the language arts standards.

The language standards provide opportunities for students to develop their vocabularies through conversation, direct instruction, and reading. The standards emphasize word meanings and nuances of words, steadily expanding students' repertoires of words and phrases. Thus students build an ability to communicate with greater precision and complexity.

Oral uses of language are common in postsecondary education and the workplace. The speaking and listening standards require students to gain, evaluate, and present complex information, ideas and evidence through speaking and listening. An important focus of the speaking and listening standards is discussion of academic topics in one-on-one, small-group and whole-class settings. Formal presentations are valued. But so are informal discussions in which students collaborate to answer questions, build understanding, and solve problems.

Summary of the proposed mathematics standards

The mathematics standards stress both procedural skills and conceptual understanding. This ensures that students absorb the critical information they need to succeed at higher levels. When students have deep understanding, they do not need prior instruction to be taught again the following year.

The mathematics standards seek to develop the following types of expertise: 1) make sense of problems and persevere in solving them; 2) reason abstractly and quantitatively; 3) construct viable arguments and critique the reasoning of others; 4) apply mathematics to solve problems in everyday life; 5) use appropriate tools strategically; 6) pay attention to precision; 7) look for and make use of structure; and 8) notice repeated calculations and look for general methods and shortcuts.

In kindergarten, the standards follow successful international models and recommendations by focusing on the number core: how numbers correspond to quantities and how to put together numbers and take them apart (the beginnings of addition and subtraction).

The standards for kindergarten to grade 5 provide students with a solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions, and decimals. With this foundation, students in kindergarten to grade 5 can do hands-on learning in geometry, algebra, probability and statistics.

The middle school standards are robust and provide a coherent and rich preparation for high school mathematics. Students who have mastered the mathematics skills through grade 7 will be well-prepared for algebraic concepts in grade 8.

The high school standards set a rigorous definition of readiness for postsecondary education and careers. Students develop a depth of understanding and the ability to apply mathematics to new situations, as college students and employees do.

Please note

This document will be updated to be responsive to questions from the public. This version was posted online on Jan. 19, 2012.

Presentation developed by the Department of Education &Early Development

January 2012

Need to Know

The department has proposed new standards for reading, writing, and mathematics

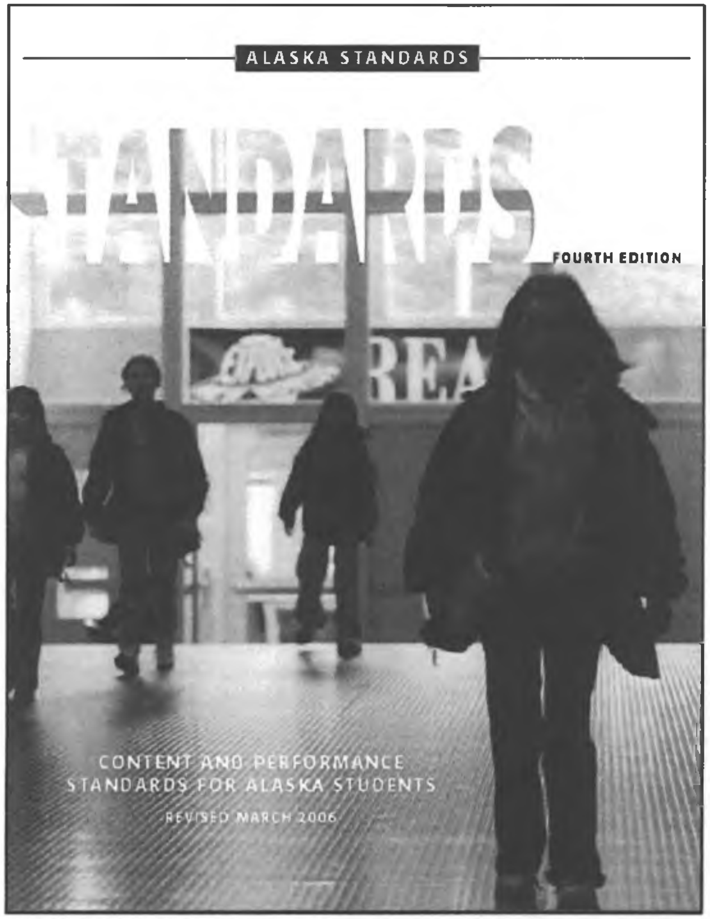
- Who participated in the development of the proposed standards?
- What is included in the proposed standards for Alaska?
- What is the timeline?
- Where is information for districts?
- Who are the contacts to support districts?

History of Alaska's Standards

1990s: Alaska standards in reading, writing, and mathematics were developed by age spans (K-2, 3-5, 6-8, and 9-12)

2004: Grade Level Expectations (GLEs) in reading, writing, and mathematics were developed to further define standards at each grade level (grades 3 – 10)

2006: Grade Level Expectations were expanded to include kindergarten through second grade



The content in red is expected to change with the new proposed standards.

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June – November 2011, Alaska educators along with national experts shared their knowledge and assisted with the revision to create the proposed standards.

The proposed Alaska standards:

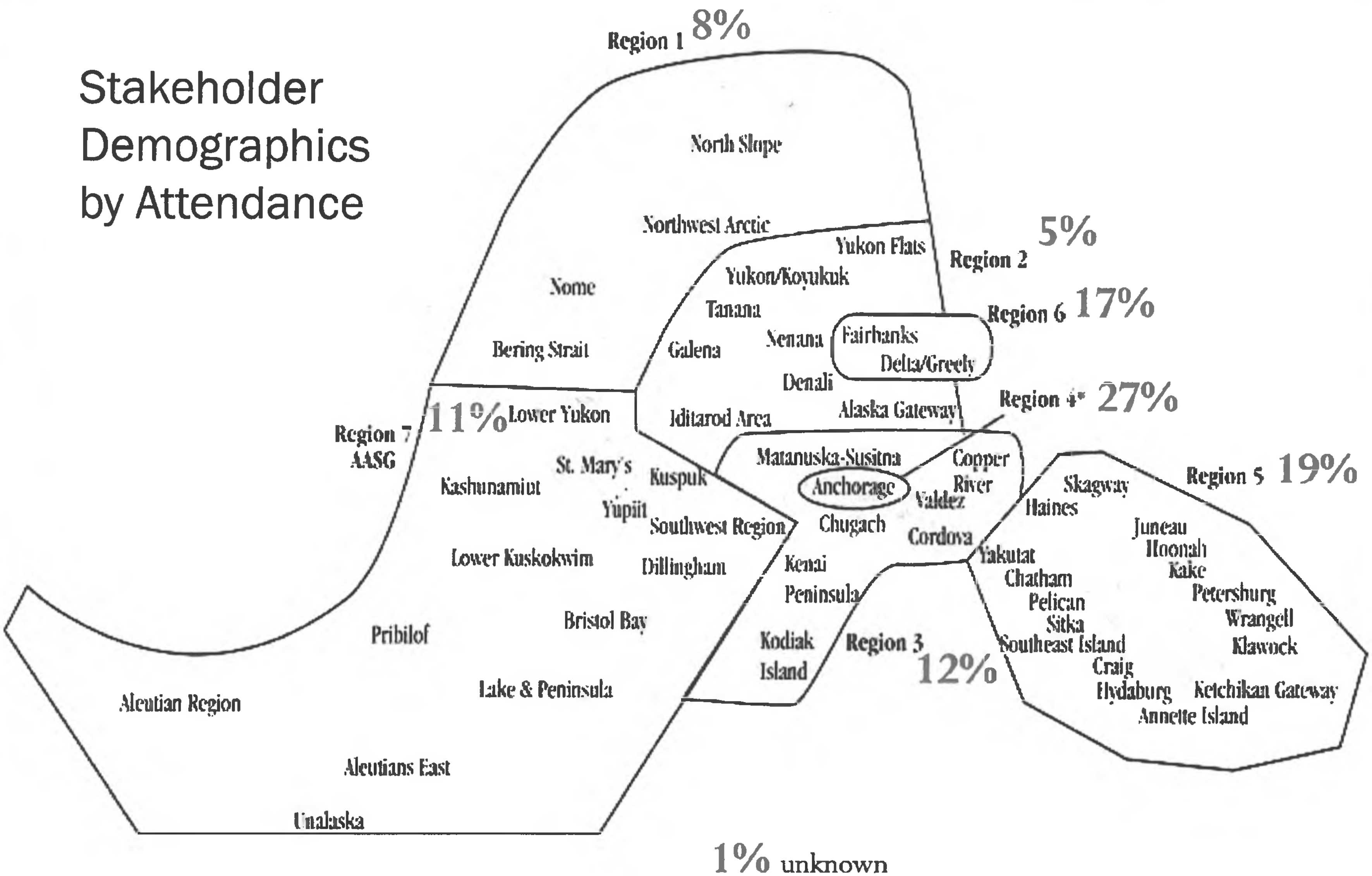
1. align with college and work readiness.
2. include rigorous content.
3. build upon strength and lessons of the GLEs.
4. outline instructional expectations for grades K-12.
5. equal in rigor to national common standards.
6. relate to real world application.

Alaska Stakeholders

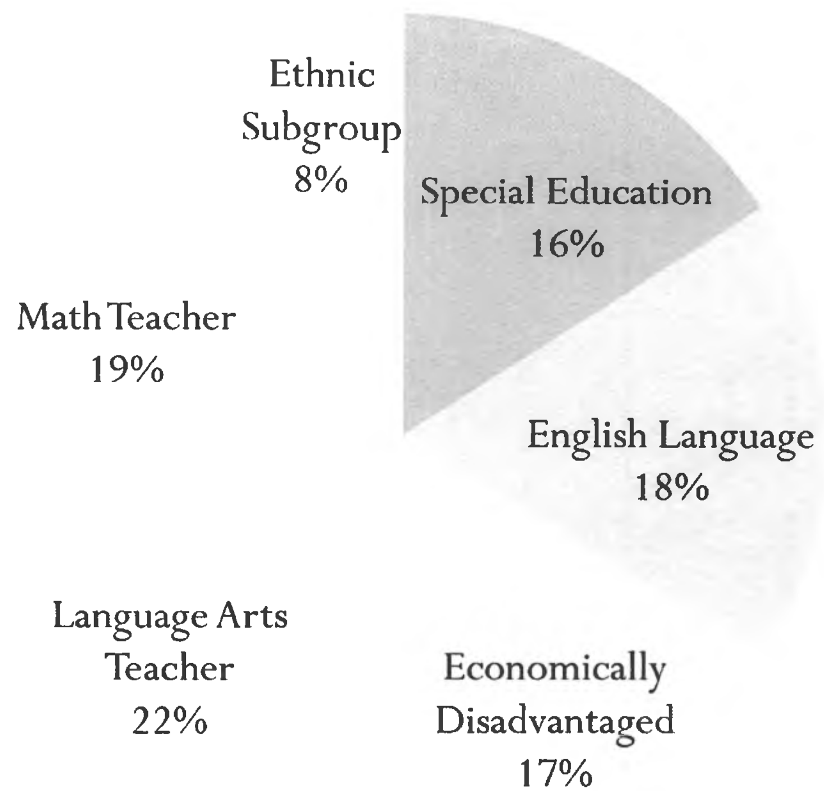
What was the representation of the stakeholders?

- Classroom teachers in reading, writing, and mathematics – kindergarten through high school
- University instructors representing multiple content areas
- Career and technical education instructors
- Alaska industry and business representatives
- District administrators
- Educators representing students with disabilities, English language learners, economically disadvantaged, and ethnic groups

Stakeholder Demographics by Attendance



Stakeholder Demographic Educational Emphasis



Tools for Alaska's Graduates

1. Increased text complexity will bridge the gap between high school and college/work readiness.
2. Connections between writing and reading will strengthen research skills.
3. Speaking and listening standards promote collaboration and clear communication.
4. Rigorous math content will have application to real world problems.
5. In-depth analysis of mathematical and logical arguments will increase critical thinking skills.

Relevancy

The new proposed standards align with the uniqueness of Alaska. Combined with the Cultural Standards and local educators, relevant learning experiences will continue.

- Same relevancy as the GLEs
- Allow for relevancy at a greater level

The proposed standards establish a strong foundation of knowledge and skills all students need for success after graduation. It is up to schools and teachers to decide how to put the standards into practice and incorporate other state standards, including the cultural standards.

Timeline

January –May 12, 2012	Public comment period
February– April 2012	Webinars and presentations about the new standards scheduled
June 2012	New standards potentially adopted
Fall 2012 – Spring 2015	Transition from GLEs to new standards
Spring 2016	New assessments potentially administered

Support for Districts

EED Website provides information and updated weekly

http://www.eed.state.ak.us/tls/assessment/GLEHome.html

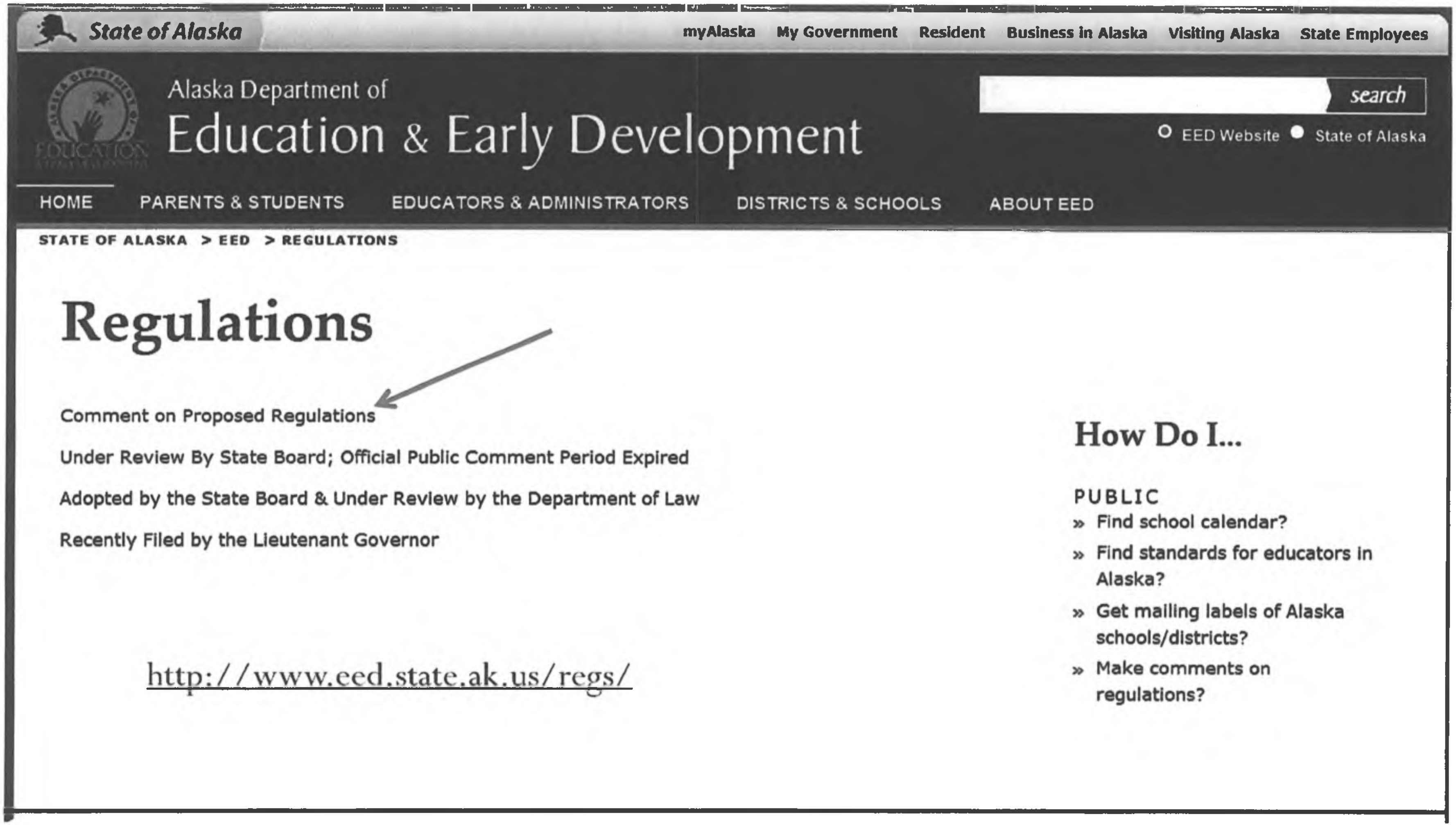
Frequently asked questions have been created for distribution http://www.eed.state.ak.us/standfaqs.html

Webinars have been scheduled for an in-depth review of the proposed standards (dates are posted on the website)

Publications for parents, teachers, and the community will be available.

Curriculum and Alignment Institute will support districts with the transition of curriculum.

Public Comment



Department Contacts

Assessment Administrator: Janet Valentour, janet.valentour@alaska.gov

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Mathematics Content Specialist: Cecilia Miller, cecilia.miller@alaska.gov

NAEP Coordinator: Jeanne Foy, jeanne.foy@alaska.gov

Department of Education
& Early Development

ALASKA MATHEMATICS STANDARDS

Pending approval by the State Board of Education in June 2012

Table of Contents

Introduction

High academic standards are an important first step in ensuring that all of Alaska's students have the tools they need for success. These standards reflect the collaborative work of Alaska educators and experts in the education community, and are also informed by public comments. Alaska teachers have played a key role in this effort, ensuring that the standards reflect the realities of the classroom. Since work began more than a year ago, the standards have undergone a thoughtful and rigorous drafting and refining process.

These standards do not tell teachers how to teach, nor do they attempt to override the unique qualities of each student and classroom. They simply establish a strong foundation of knowledge and skills all students need for success after graduation. It is up to schools and teachers to decide how to put the standards into practice and incorporate other state standards, including the cultural standards.

Description of Mathematics Standards

The standards stress procedural skills and conceptual understanding; this ensures students are learning and absorbing the critical information they need to succeed at higher levels. Often current practices suggest students learn enough to get by on the next test, only to forget the information later. This practice creates the need for prior instruction to be taught again the following year.

In kindergarten, the standards follow successful international models and recommendations by focusing kindergarten work on the number core: learning how numbers correspond to quantities, and learning how to put numbers together and take them apart (the beginnings of addition and subtraction).

The K-5 standards provide students with a solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions and decimals--which help young students build the foundation to successfully apply more demanding math concepts and procedures, and move into applications.

Having built a strong foundation, K-5 students can do hands-on learning in geometry, algebra and probability and statistics. Students who have completed 7th grade and mastered the content and skills through the 7th grade will be well-prepared for algebra in grade 8.

The middle school standards are robust and provide a coherent and rich preparation for high school mathematics.

The high school standards set a rigorous definition of readiness by helping students develop a depth of understanding and ability to apply mathematics to novel situations, as college students and employees regularly do.

Alaska Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Each Standard for Mathematical Practice listed below is followed by a set of grade-span descriptors. These descriptors of the Standards of Mathematical Practice are meant to help students, parents and educators to picture how these practices might be demonstrated by students and should not serve as a checklist. Within the grade span, students should apply the practices using specific grade-level content. Additionally, students at higher grade spans may revisit earlier grade-span proficiencies as the rigor of the content increases.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

In grades K-2 mathematically proficient students will:

- focus on the problem and check for alternate methods
- check if the solution makes sense

In grades 3-5 mathematically proficient students will:

- explain correspondences between equations, verbal descriptions, tables, and graphs
- draw diagrams of important features and relationships, graph data, and search for regularity or trends
- use concrete objects or pictures to help conceptualize and solve a problem
- understand the approaches of others to solving complex problems
- identify correspondences between different approaches
- check if the solution makes sense

In grades 6-8 mathematically proficient students will:

- explain correspondences between a new problem and previous problems
- represent algebraic expressions numerically, graphically, concretely/with manipulatives, verbally/written
- explain connections between the multiple representations
- determine the question that needs to be answered
- make a plan for attempting a problem
- choose a reasonable strategy
- identify the knowns and unknowns in a problem
- use previous knowledge and skills to simplify and solve problems
- break a problem into manageable parts or simpler problems
- solve a problem in more than one way

In grades 9-12 mathematically proficient students will:

- make connections between a new problem and previous problems
- determine the question that needs to be answered
- make a plan for attempting a problem
- choose a reasonable strategy
- identify the knowns and unknowns in a problem
- use previous knowledge and skills to simplify and solve problems
- break a problem into manageable parts or simpler problems
- represent algebraic expressions numerically, graphically, concretely/with manipulatives, verbally/written
- explain connections between the multiple representations
- solve a problem in more than one way
- explain the meaning of a problem and look for an entry point
- analyze a problem and make a plan for solving it
- explain correspondence between differing approaches to identify regularity and trends
- check answer using a different method
- identify correspondence between different approaches
- monitor and evaluate progress and change course if necessary
- check the answers to problems using a different method and continually ask, "Does this make sense?"

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

In grades K-2 mathematically proficient students will:

- represent a situation symbolically and/or with manipulatives
- create a coherent representation of the problem
- use units of measurement consistently

In grades 3-5 mathematically proficient students will:

- represent a situation symbolically
- create a coherent representation of the problem
  have the ability to show how problem has a realistic meaning
- reflect during the manipulation process in order to probe into the meanings for the symbols involved
- use units consistently

In grades 6-8 mathematically proficient students will:

- represent a situation symbolically and carry out its operations
- create a coherent representation of the problem
- translate an algebraic problem to a real world context
- explain the relationship between the symbolic abstraction and the context of the problem
- compute using different properties
- consider the quantitative values, including units, for the numbers in a problem

In grades 9-12 mathematically proficient students will:

- decontextualize to abstract a given situation and represent it symbolically and manipulate the representing symbols.
- reflect during the manipulation process in order to probe into the meanings for the symbols involved
- create a coherent representation of the problem
- make sense of quantities and their relationships in problem situations
- attend to the meanings of quantities
- use flexibility with different properties of operations and objects
- translate an algebraic problem to a real world context
- explain the relationship between the symbolic abstraction and the context of the problem
- compute using different properties
- consider the quantitative values, including units, for the numbers in a problem

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

In grades K-2 mathematically proficient students will:

- construct arguments using concrete referents such as objects, drawings, diagrams, and actions
- justify conclusions, communicate conclusions
- listen to arguments and decide whether the arguments make sense

In grades 3-5 mathematically proficient students will:

- construct arguments using concrete referents such as objects, drawings, diagrams, and actions
- justify conclusions, communicate conclusions, listen and respond to arguments, decide whether the argument makes sense, and ask questions to clarify the argument
- reason inductively about data, making plausible arguments that take into account the context from which the data arose

In grades 6-8 mathematically proficient students will:

- construct arguments using both concrete and abstract explanations
- justify conclusions, communicate conclusions, and respond to the arguments
- listen to arguments, critique their viability, and ask questions to clarify the argument
- compare effectiveness of two arguments by identifying and explaining both logical and/or flawed reasoning
- recognize general mathematical truths and use statements to justify the conjectures
- identify special cases or counter-examples that don't follow the mathematical rules

- infer meaning from data and make arguments using its context

In grades 9-12 mathematically proficient students will:

- construct arguments using both concrete and abstract explanations
- justify conclusions in a variety of ways, communicate the methodology, and respond to the arguments
- reason inductively about data and make plausible arguments that take into account the context from which the data arose
- understand and use stated assumptions, definitions, and previously established results in constructing arguments
- make conjectures and build a logical progression of statements to explore the truth of the conjectures
- analyze situations by breaking them into cases and recognize and use counter-examples
- recognize general mathematical truths and statements to justify the conjectures
- identify special cases or counter-examples that don't follow the mathematical rules
- infer meaning from data and make arguments using its context
- compare effectiveness of two arguments by identifying and explaining both logical and/or flawed reasoning

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

In grades K-2 mathematically proficient students will:

- apply mathematics to solve problems in everyday life
- identify important quantities in a practical situation and model the situation with manipulatives or pictures
- interpret mathematical results in the context of the situation and reflect on whether the results make sense

In grades 3-5 mathematically proficient students will:

- apply mathematics to solve problems arising in everyday life
- identify important quantities in a practical situation and model the situation using such tools as manipulatives, diagrams, two-way tables, graphs or pictures
- interpret mathematical results in the context of the situation and reflect on whether the results make sense
- apply mathematical knowledge, make assumptions and approximations to simplify a complicated situation

In grades 6-8 mathematically proficient students will:

- apply mathematics to solve problems arising in everyday life and society
- identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, and formulas
- interpret their mathematical results in the context of the situation and reflect on whether the results make sense
- make assumptions and approximations to simplify a situation, realizing the final solution will need to be revised
- analyze quantitative relationships to draw conclusions
- reflect on whether their results make sense
- improve the model if it has not served its purpose

In grades 9-12 mathematically proficient students will:

- apply mathematics to solve problems in everyday life, society, and workplace
- identify important quantities in a practical situation and map the relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas
- consistently interpret mathematical results in the context of the situation and reflect on whether the results make sense
- apply knowledge, making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later
- make assumptions and approximations to simplify a situation, realizing the final solution will need to be revised
- identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, and formulas
- analyze quantitative relationships to draw conclusions
- improve the model if it has not served its purpose

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

In grades K-2 mathematically proficient students will:

- select the available tools (such as pencil and paper, manipulatives, rulers, and available technology) when solving a mathematical problem
- be familiar with tools appropriate for the grade level to make sound decisions about when each of these tools might be helpful
- identify relevant external mathematical resources and use them to pose or solve problems
- use technological tools to explore and deepen their understanding of concepts

In grades 3-5 mathematically proficient students will:

- select the available tools (such as pencil and paper, manipulatives, rulers, calculators, a spreadsheet, and available technology) when solving a mathematical problem
- be familiar with tools appropriate for their grade level to make sound decisions about when each of these tools might be helpful
- identify relevant external mathematical resources and use them to pose or solve problems
- use technological tools to explore and deepen their understanding of concepts
- detect possible errors by strategically using estimation and other mathematical knowledge
- know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data

In grades 6-8 mathematically proficient students will:

- select and use tools appropriate to the task: pencil and paper, protractor, visual and physical fraction models, algebra tiles, geometric models, calculator, spreadsheet, and interactive geometry software.
- use estimation and other mathematical knowledge to confirm the accuracy of their problem solving
- identify relevant external and digital mathematical resources and use them to pose or solve problems
- represent and compare possibilities visually with technology when solving a problem
- explore and deepen their understanding of concepts through the use of technological tools

In grades 9-12 mathematically proficient students will:

- select and accurately use appropriate, available tools (such as pencil and paper, concrete or virtual manipulatives such as geoboards and algebra tiles, graphing and simpler calculators, a spreadsheet, and available technology) when solving a mathematical problem
- identify relevant external and digital mathematical resources and use the resources to pose or solve problems
- detect possible errors by strategically using estimation and other mathematical knowledge
- use technology to visualize the results of varying assumptions, exploring consequences, comparing predictions with data, and deepening understanding of concepts

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

In grades K-2 mathematically proficient students will:

- give thoughtful explanations to each other
- use clear definitions and reasoning in discussion with others
- state the meaning of symbols they choose, including using the equal sign consistently and appropriately

In grades 3-5 mathematically proficient students will:

- give carefully formulated explanations to each other

- use clear definitions and reasoning in discussion with others
- state the meaning of symbols, including using the equal sign consistently and appropriately
- specify units of measure, and label axes to clarify the correspondence with quantities in a problem
- calculate accurately and efficiently
- express numerical answers with a degree of precision appropriate for the problem context

In grades 6-8 mathematically proficient students will:

- use clear definitions in explanations
- understand and use specific symbols accurately and consistently: equality, inequality, ratios, parenthesis, for multiplication and division, absolute value, square root
- specify units of measure, and label axes to clarify the correspondence with quantities in a problem
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context

In grades 9-12 mathematically proficient students will:

- communicate precisely to others
- use clear definitions in explanations
- use symbols consistently and appropriately
- specify units of measure, and label axes to clarify the correspondence with quantities in a problem
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context
- examine claims and make explicit use of definitions

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

In all grade levels mathematically proficient students will:

- discern a pattern or structure
- understand complex structures as single objects or as being composed of several objects
- check if the answer is reasonable

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

In all grade levels mathematically proficient students will:

- identify if calculations or processes are repeated
- use alternative and traditional methods to solve problems
- evaluate the reasonableness of their intermediate results, while attending to the details

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Coding for K-8 Mathematical Domains:

1. Counting, Cardinality, and Ordinality – CC
2. Operations and Algebraic Thinking – OA
3. Number and Operations in Base Ten – NBT
4. Measurement and Data - MD
5. Number and Operations—Fractions - NF
6. Geometry - G
7. Ratios and Proportional Relationships – RP
8. The Number System - NS
9. Expressions and Equations - EE
10. Functions - F
11. Statistics and Probability - SP

Instructional Focus: Kindergarten through Second Grade

Kindergarten	Grade 1	Grade 2
In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics. (1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away. (2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary.	In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes. (1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20.	In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. (1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones). (2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to

Kindergarten	Grade 1	Grade 2
They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.	By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction. (2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes. (3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1] (4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they	mentally calculate sums and differences for numbers with only tens or only hundreds. (3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length. (4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Kindergarten	Grade 1	Grade 2
	are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry [1]Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.	

Grade K	Grade 1	Grade 2
Counting and Cardinality K.CC **Know number names and the count sequence.** K.CC.1. Count to 100 by ones and by tens. K.CC.2. Count forward beginning from a given number within the known sequence. K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). **Count to tell the number of objects.** K.CC.4. Understand the relationship between numbers and quantities to cardinality. a. When counting objects, say the number names in standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	**Counting, Cardinality, and Ordinality 1.CC** **Skip count and know ordinal names.** 1.CC.1. Skip count by 2s and 5s. 1.CC.2. Use ordinal numbers correctly when identifying object position (e.g., first, second, third, etc.). **Count to tell the number of objects.** 1.CC.3. Count a large quantity of objects by grouping into 10s and counting by 10s and 1s to find the quantity.	

Grade K	Grade 1	Grade 2
K.CC.5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.		
Compare numbers. K.CC.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (e.g., by using matching, counting, or estimating strategies).	**Compare numbers.** 1.CC.4. Use the symbols for greater than, less than or equal to when comparing two numbers or groups of objects. 1.CC.5. Order numbers from 1-100.	
K.CC.7. Compare and order two numbers between 1 and 10 presented as written numerals.	1.CC.6. Estimate how many and how much in a given set to 20 and then verify estimate by counting.	
<u>**Operations and Algebraic Thinking K.OA**</u> **Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps) acting out situations, verbal explanations, expressions, or equations.	<u>**Operations and Algebraic Thinking 1.OA**</u> **Represent and solve problems involving addition and subtraction.** 1.OA.1. Use addition and subtraction strategies to solve word problems (using numbers up to 20), involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, using a number line(e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem.	<u>**Operations and Algebraic Thinking 2.OA**</u> **Represent and solve problems involving addition and subtraction.** 2.OA.1. Use addition and subtraction strategies to estimate, then solve one- and two-step word problems (using numbers up to 100) involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions (e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem.

Grade K	Grade 1	Grade 2
K.OA.2. Add or subtract whole numbers to 10 (e.g., by using objects or drawings to solve word problems). K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way (e.g., by using objects or drawings, and record each decomposition by a drawing or equation). K.OA.4. For any number from 1-4, find the number that makes 5 when added to the given number and, for any number from 1-9, find the number that makes 10 when added to the given number (e.g., by using objects, drawings or 10 frames) and record the answer with a drawing or equation. K.OA.5. Fluently add and subtract numbers up to 5.	1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 (e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem. **Understand and apply properties of operations and the relationship between addition and subtraction.** 1.OA.3. Apply properties of operations as strategies to add and subtract. (Students need not know the name of the property.) Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (Associative property of addition.) Demonstrate that when adding or subtracting zero to any number, the quantity does not change (Identity property of addition.) 1.OA.4. Understand subtraction as an unknown-addend problem. *For example, subtract 10-8 by finding the number that makes 10 when added to 8.*	

Grade K	Grade 1	Grade 2
	Add and subtract using numbers up to 20. 1.OA.5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). 1.OA.6. Add and subtract using numbers up to 20. Use strategies such as • counting on • making ten (8 + 6 = 8 + 2 +4 = 10 + 4 = 14) • decomposing a number leading to a ten (13 - 4 = 13 -3 -1 =10 -1 = 9) • using the relationship between addition and subtraction, such as fact families, (8 + 4 = 12 and 12 - 8 = 4) • creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	**Add and subtract using numbers up to 20.** 2.OA.2. Fluently add and subtract using numbers up to 20 using mental strategies. Know from memory all sums of two-digit numbers.
	Work with addition and subtraction equations. 1.OA.7. Understand the meaning of the equal sign (e.g., read equal sign as "same as") and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2)* 1.OA.8. Determine the unknown whole number in an addition or subtraction equation. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 6 + 6 = ?, 5 = ? - 3.*	**Work with equal groups of objects to gain foundations for multiplication.** 2.OA.3. Determine whether a group of objects (up to 20) is odd or even (e.g., by pairing objects and comparing, counting by 2s). Model an even number as two equal groups of objects and then write an equation as a sum of two equal addends. 2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns. Write an equation to express the total as repeated addition (e.g., array of 4 by 5 would be 5 + 5 + 5 + 5 = 20).

Grade K	Grade 1	Grade 2
Identify and continue patterns. K.OA.6. Recognize, identify and continue simple patterns of color, shape, and size. **<u>Number and Operations in Base Ten K.NBT</u>** **Work with numbers 11-19 to gain foundations for place value.** K.NBT.1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones (e.g., by using objects or drawings) and record each composition and decomposition by a drawing or equation (e.g., 18=10 +8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight or nine ones.	**Identify and continue patterns.** 1.OA.9. Identify, continue and label patterns (e.g., aabb, abab). Create patterns using number, shape, size, rhythm or color. **<u>Number and Operations in Base Ten 1.NBT</u>** **Extend the counting sequence.** 1.NBT.1. Count to 120. In this range, read, write and order numerals and represent a number of objects with a written numeral. **Understand place value.** 1.NBT.2. Model and identify place value positions of two digit numbers. Include: a. 10 can be thought of as a bundle of ten ones, called a "ten". b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90, refer to one, two, three, four, five, six, seven, eight or nine tens (and 0 ones). 1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, <.	**Identify and continue patterns.** 2.OA.5. Identify, continue and label number patterns (e.g., aabb, abab). Describe a rule that determines and continues a sequence or pattern. **<u>Number and Operations in Base Ten 2.NBT</u>** **Understand place value.** 2.NBT.1. Model and identify place value positions of three digit numbers. Include: a. 100 can be thought of as a bundle of ten tens --called a "hundred". b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). 2.NBT.2. Count up to 1000, skip-count by 5s, 10s and 100s. 2.NBT.3. Read, write, order up to 1000 using base-ten numerals, number names and expanded form. 2.NBT.4. Compare two three-digit numbers based on the meanings of the hundreds, tens and ones digits, using >, =, < symbols to record the results.

Grade K	Grade 1	Grade 2
	Use place value understanding and properties of operations to add and subtract. 1.NBT.4. Add using numbers up to 100 including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10. Use: • concrete models or drawings and strategies based on place value • properties of operations • and/or relationship between addition and subtraction; Relate the strategy to a written method and explain the reasoning used. Demonstrate in adding two-digit numbers, tens and tens are added, ones and ones are added and sometimes it is necessary to compose a ten from ten ones. 1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number without having to count. 1.NBT.6. Subtract multiples of 10 up to 100. Use: • concrete models or drawings • strategies based on place value • properties of operations • and/or the relationship between addition and subtraction Relate the strategy to a written method and explain the reasoning used.	**Use place value understanding and properties of operations to add and subtract.** 2.NBT.5. Fluently add and subtract using numbers up to 100. Use: • strategies based on place value • properties of operations • and/or the relationship between addition and subtraction. 2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations. 2.NBT.7. Add and subtract using numbers up to 1000. Use: • concrete models or drawings and strategies based on place value • properties of operations • and/or relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used. Demonstrate in adding or subtracting three-digit numbers, hundreds and hundreds are added or subtracted, tens and tens are added or subtracted, ones and ones are added or subtracted and sometimes it is necessary to compose a ten from ten ones or a hundred from ten tens.

Grade K	Grade 1	Grade 2
		2.NBT.8. Mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number. 2.NBT.9. Explain or illustrate the processes of addition or subtraction and their relationship using place value and the properties of operations.
Measurement and Data K.MD **Describe and compare measurable attributes.** K.MD.1. Describe measurable attributes of objects (e.g., length or weight). Match measuring tools to attribute (e.g., ruler to length). Describe several measureable attributes of a single object. K.MD.2. Make comparisons between two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* **Classify objects and count the number of objects in each category.** K.MD.3. Classify objects into given categories (attribute). Count the number of objects in each category (limit category counts to be less than or equal to 10).	**Measurement and Data 1.MD** **Measure lengths indirectly and by iterating length units.** 1.MD.1. Measure and compare three objects using standard or non-standard units. 1.MD.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.	**Measurement and Data 2.MD** **Measure and estimate lengths in standard units.** 2.MD.1. Measure the length of an object by selecting and using standard tools such as rulers, yardsticks, meter sticks, and measuring tapes. 2.MD.2. Measure the length of an object twice using different length units for the two measurements. Describe how the two measurements relate to the size of the unit chosen. 2.MD.3. Estimate, measure and draw lengths using whole units of inches, feet, yards, centimeters and meters. 2.MD.4. Measure to compare lengths of two objects, expressing the difference in terms of a standard length unit. **Relate addition and subtraction to length.** 2.MD.5. Solve addition and subtraction word problems using numbers up to 100 involving length that are given in the same units (e.g., by using drawings of rulers). Write an equation with a symbol for the unknown to represent the problem.

Grade K	Grade 1	Grade 2
Work with time and money. K.MD.4. Name in sequence the names of the week. K.MD.5 Tell time to the hour using both analog and digital clocks. K.MD.6. Identify coins by name.	**Tell and write time and work with money.** 1.MD.3. Tell and write time in hours and half hours using both analog and digital clocks. 1.MD.4. Read a calendar distinguishing yesterday, today and tomorrow. Read and write a date. 1.MD.5. Recognize and read money symbols including $ and ¢. 1.MD.6. Identify values of coins (e.g., nickel = 5 cents, quarter = 25 cents). Identify equivalent values of coins up to $1 (e.g., 5 pennies = 1 nickel, 5 nickels = 1 quarter).	**Work with time and money.** 2.MD.6. Tell and write time to the nearest five minutes using a.m. and p.m. from analog and digital clocks. 2.MD.7. Solve word problems involving dollar bills and coins using the $ and ¢ symbols appropriately.
	Represent and interpret data. 1.MD.7. Organize, represent and interpret data with up to three categories. Ask and answer comparison and quantity questions about the data.	**Represent and interpret data.** 2.MD.8. Collect, record, interpret, represent, and describe data in a table, graph or plot. 2.MD.9. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart and compare problems using information presented in a bar graph.

Grade K	Grade 1	Grade 2
<u>**Geometry K.G**</u>	<u>**Geometry 1.G**</u>	<u>**Geometry 2.G**</u>
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	**Reason with shapes and their attributes.**	**Reason with shapes and their attributes.**
K.G.1. Describe objects in the environment using names of shapes and describe their relative positions (e.g., *above*, *below*, *beside*, *in front of*, *behind*, *next to)*.	1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes. Identify shapes that have non-defining attribute (e.g., color, orientation, overall size). Build and draw shapes given specified attributes.	2.G.1. Identify and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces compared visually, not by measuring. Identify triangles, quadrilaterals, pentagons, hexagons and cubes.
K.G.2. Name shapes regardless of their orientation or overall size.	1.G.2. Compose (put together) two-dimensional or three-dimensional shapes to form larger (composite) shapes. Compose new shapes from the composite shapes.	2.G.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
K.G.3. Identify shapes as two-dimensional or three-dimensional.	1.G.3. Partition circles and rectangles into two and four equal parts. Describe the parts using the words, *halves, fourths*, and *quarters* and phrases *half of, fourth of* and *quarter of.* Describe the whole as two of or four of the parts. Understand for these examples that decomposing (break apart) into more equal shares creates smaller shares.	2.G.3. Partition circles and rectangles into parts, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
Analyze, compare, create, and compose shapes. K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices), and other attributes (e.g., having sides of equal lengths).		
K.G.5. Build shapes (e.g., using sticks and clay) and draw shapes.		
K.G.6. Put together two-dimensional shapes to form larger shapes (e.g., join two triangles with full sides touching to make a rectangle).		

Instructional Focus: Third Grade though Fifth Grade

Grade 3	Grade 4	Grade 5
In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes. (1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division. (2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger	In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. (1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures	In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume. (1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.) (2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to

Grade 3	Grade 4	Grade 5
bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators. (3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle. (4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.	to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context. (2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number. (3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.	add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately. (3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Grade 3	Grade 4	Grade 5
Operations and Algebraic Thinking 3.OA **Represent and solve problems involving multiplication and division.** 3.OA.1. Interpret products of whole numbers (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each). *For example, show objects in rectangular arrays or describe a context in which a total number of objects can be expressed as 5 × 7.* 3.OA.2. Interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each). *For example, deconstruct rectangular arrays or describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* 3.OA.3. Use multiplication and division numbers up to 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	**Operations and Algebraic Thinking 4.OA** **Use the four operations with whole numbers to solve problems.** 4.OA.1. Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 groups of 7 and 7 groups of 5). (Commutative property) Represent verbal statements of multiplicative comparisons as multiplication equations. 4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem or missing numbers in an array). Distinguish multiplicative comparison from additive comparison. 4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	**Operations and Algebraic Thinking 5.OA** **Write and interpret numerical expressions.** 5.OA.1. Use parentheses to construct numerical expressions, and evaluate numerical expressions with these symbols.

Grade 3	Grade 4	Grade 5
3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers (e.g., $8 \times ? = 48$, $5 = ? \div 3$, $6 \times 6 = ?$, and find $32 \div 8$ by finding the number that makes 32 when multiplied by 8).		
Understand properties of multiplication and the relationship between multiplication and division. 3.OA.5. Make, test, support, draw conclusions and justify conjectures about properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) • Commutative property of multiplication) If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. • Associative property of multiplication $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. • Distributive property) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. • Inverse property (relationship) of multiplication and division.	**Gain familiarity with factors and multiples.** 4.OA.4. • Find all factor pairs for a whole number in the range 1–100. • Explain the correlation/differences between multiples and factors. • Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. • Determine whether a given whole number in the range 1–100 is prime or composite.	

Grade 3	Grade 4	Grade 5
Multiply and divide within 100. 3.OA.6. Fluently multiply and divide numbers up to 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.** 3.OA.7. Solve and create two-step word problems using the any of the four operations. Represent these problems using equations with a symbol (box, circle, question mark) standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 3.OA.8. Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	**Generate and analyze patterns.** 4.OA.5. Generate a number or shape pattern, table, t-chart, input/output function that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. Be able to express the pattern in algebraic terms. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* 4.OA.6 Extend patterns that use addition, subtraction, multiplication, division or symbols, up to 10 terms, represented by models (function machines), tables, sequences, or in problem situations (L)	**Analyze patterns and relationships.** 5.OA.2. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Grade 3	Grade 4	Grade 5
Number and Operations in Base Ten 3.NBT	**Number and Operations in Base Ten 4.NBT**	**Number and Operations in Base Ten 5.NBT**
Use place value understanding and properties of operations to perform multi-digit arithmetic.	**Generalize place value understanding for multi-digit whole numbers.**	**Understand the place value system.**
3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100.	4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	5.NBT.1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
3.NBT.2. Use strategies and/or algorithms to fluently add and subtract with numbers up to 1000, demonstrating understanding of place value, properties of operations, and/or the relationship between addition and subtraction.	4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on the value of the digits in each place, using >, =, and < symbols to record the results of comparisons.	5.NBT.2. Explain and extend the patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain and extend the patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3.NBT.3. Multiply one digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 10 x 60) using strategies based on place value and properties of operations.	4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place using a variety of estimation methods; be able to describe, compare, and contrast solutions.	5.NBT.3. Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form [e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 (1/10) + 9 (1/100) + 2 (1/1000)]. b. Compare two decimals to thousandths place based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
	Use place value understanding and properties of operations to perform multi-digit arithmetic. 4.NBT.4. Fluently add and subtract multi-digit whole numbers using any algorithm. Verify the reasonableness of the results.	

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	4.NBT.5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.NBT.6. Find whole-number quotients and remainders with one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	5.NBT.4. Use place values understanding to round decimals to any place. **Perform operations with multi-digit whole numbers and with decimals to hundredths.** 5.NBT.5. Fluently multiply multi-digit whole numbers using a standard algorithm. 5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, number lines, real life situations, and/or area models. 5NBT.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between the operations. Explain their reasoning in getting their answers.

Grade 3	Grade 4	Grade 5
<u>Number and Operations—Fractions 3.NF</u> **(limited in this grade to fractions with denominators 2, 3, 4, 6, and 8)** **Develop understanding of fractions as numbers.** 3.NF.1. Understand a fraction $1/b$ (e.g., 1/4) as the quantity formed by 1 part when a whole is partitioned into b (e.g., 4) equal parts; understand a fraction a/b (e.g., 2/4) as the quantity formed by a (e.g., 2) parts of size $1/b$. (e.g., 1/4) 3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction $1/b$ (e.g., 1/4) on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b (e.g., 4) equal parts. Recognize that each part has size $1/b$ (e.g., 1/4) and that the endpoint of the part based at 0 locates the number $1/b$ (e.g., 1/4) on the number line. b. Represent a fraction a/b (e.g., 2/8) on a number line diagram or ruler by marking off a lengths $1/b$ (e.g., 1/8) from 0. Recognize that the resulting interval has size a/b (e.g., 2/8) and that its endpoint locates the number a/b (e.g., 2/8) on the number line.	**<u>Number and Operations—Fractions 4.NF</u>** **(limited in this grade to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100)** **Extend understanding of fraction equivalence and ordering.** 4.NF.1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 4.NF.2. Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).	**<u>Number and Operations—Fractions 5.NF</u>** **Use equivalent fractions as a strategy to add and subtract fractions.** 5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. *(In general,* $a/b + c/d = (ad + bc)/bd$.*)* 5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and check the reasonableness of answers. *For example, recognize an incorrect result* $2/5 + 1/2 = 3/7$, *by observing that* $3/7 < 1/2$.

Grade 3	Grade 4	Grade 5
3.NF.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent if they are the same size (modeled) or the same point on a number line. b. Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model). c. Express and model whole numbers as fractions, and recognize and construct fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).	**Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.** 4.NF.3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions (e.g., by using a visual fraction model). *Examples: 3/8 = 1/8 + 1/8 + 1/8 ;3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators (e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction). d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators (e.g., by using visual fraction models and equations to represent the problem).	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** 5.NF.3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers (e.g., by using visual fraction models or equations to represent the problem). *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* 5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Grade 3	Grade 4	Grade 5
	4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number (e.g., by using visual fraction models and equations to represent the problem). Check for the reasonableness of the answer. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	5.NF.5 Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying *a*/*b* by 1. (Division of a fraction by a fraction is not a requirement at this grade.) 5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers (e.g., by using visual fraction models or equations to represent the problem).

Grade 3	Grade 4	Grade 5
	Understand decimal notation for fractions, and compare decimal fractions. 4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* 4.NF.6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* 4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual model).	5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions (e.g., by using visual fraction models and equations to represent the problem). *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Grade 3	Grade 4	Grade 5
<u>**Measurement and Data 3.MD**</u> **Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.** 3.MD.1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes or hours (e.g., by representing the problem on a number line diagram or clock). 3.MD.2. Estimate and measure liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve and create one-step word problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem). (Excludes multiplicative comparison problems [problems involving notions of "times as much."])	<u>**Measurement and Data 4.MD**</u> **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit, and time.** 4.MD.1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36).* 4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	<u>**Measurement and Data 5.MD**</u> **Convert like measurement units within a given measurement system. Solve problems involving time.** 5. MD.1. Identify, estimate measure, and convert equivalent measures within systems English length (inches, feet, yards, miles) weight (ounces, pounds, tons) volume (fluid ounces, cups, pints, quarts, gallons) temperature (Fahrenheit) Metric length (millimeters, centimeters, meters, kilometers) volume (milliliters, liters), temperature (Celsius), (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems using appropriate tools. 5. MD.2. Solve real-world problems involving elapsed time between world time zones. (L)

Grade 3	Grade 4	Grade 5
3.MD.3. Select an appropriate unit of English, metric, or non-standard measurement to estimate the length, time, weight, or temperature (L)	4.MD.3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* 4.MD.4. Solve real-world problems involving elapsed time between U.S. time zones (including Alaska Standard time) (L)	
Represent and interpret data. 3.MD.4. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* 3.MD.5. Measure and record lengths using rulers marked with halves and fourths of an inch. Make a line plot with the data, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. 3.MD.6. Explain the classification of data from real-world problems shown in graphical representations .Use the terms minimum and maximum. (L)	**Represent and interpret data.** 4.MD.5. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* 4.MD.6. Explain the classification of data from real-world problems shown in graphical representations including the use of terms mean, range, median and mode with a given set of data. (L)	**Represent and interpret data.** 5.MD.3. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Grade 3	Grade 4	Grade 5
Geometric measurement: understand concepts of area and relate area to multiplication and to addition. 3.MD.7. Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit is said to have “one square unit” and can be used to measure area. b. Demonstrate that a plane figure can be covered without gaps or overlaps by *n* (e.g., 6) unit squares is said to have an area of *n* (e.g., 6) square units. 3.MD.8. Measure areas by tiling with unit squares (square centimeters, square meters, square inches, square feet, and improvised units).	**Geometric measurement: understand concepts of angle and measure angles.** 4.MD.7. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand the following concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. 4.MD.8. Measure and draw angles in whole-number degrees using a protractor. Estimate and sketch angles of specified measure. 4.MD.9. Recognize angle measure as additive. When an angle is divided into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems (e.g., by using an equation with a symbol for the unknown angle measure).	**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** 5.MD.4. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. 5.MD.5. Estimate and measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units.

Grade 3	Grade 4	Grade 5
3.MD.9. Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. *For example, after tiling rectangles, develop a rule for finding the area of any rectangle.* b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use area models (rectangular arrays) to represent the distributive property in mathematical reasoning. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. d. Recognize area as additive. Find areas of rectangular figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. *For example, the area of a 7 by 8 rectangle can be determined by decomposing it into a 7 by 3 rectangle and a 7 by 5 rectangle.*		5.MD.6. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Estimate and find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Demonstrate the associative property of multiplication by using the product of three whole-numbers to find volumes (length x width x height). b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two conjoined right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Grade 3	Grade 4	Grade 5
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. 3.MD.10. Solve real world and mathematical problems involving perimeters of polygons, including: • finding the perimeter given the side lengths, •finding an unknown side length, • exhibiting rectangles with the same perimeter and different areas • exhibiting rectangles with the same area and different perimeters.		
<u>Geometry 3.G</u> **Reason with shapes and their attributes.** 3.G.1. Categorize shapes by different attribute classifications and recognize that shared attributes can define a larger category. Generalize to create examples or non-examples. 3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	<u>Geometry 4.G</u> **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.** 4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular, parallel, and intersecting line segments. Identify these in plane figures. 4.G.2. Classify plane figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. 4.G.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	<u>Geometry 5.G</u> **Graph points on the coordinate plane to solve real-world and mathematical problems.** 5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

Grade 3	Grade 4	Grade 5
		5.G.2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **Classify two-dimensional figures into categories based on their properties.** 5.G.3. Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* 5.G.4. Classify two-dimensional figures in a hierarchy based on attributes and properties

Instructional Focus: Sixth Grade though Eighth Grade

Grade 6	Grade 7	Grade 8
In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking. (1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates. (2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They	In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples. (1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships. (2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction,	In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. (1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently

Grade 6	Grade 7	Grade 8
reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane. (3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities. (4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their	multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. (3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms. (4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.	implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems. (2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations. (3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances

Grade 6	Grade 7	Grade 8
variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected. Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.		between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Grade 6	Grade 7	Grade 8
<u>Ratios and Proportional Relationships 6.RP</u> **Understand ratio concepts and use ratio reasoning to solve problems.** 6.RP.1. Write and describe the relationship in real life context between two quantities using ratio language. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* 6.RP.2. Understand the concept of a unit rate (a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship) and apply it to solve real world problems (e.g., unit pricing, constant speed). *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*	**<u>Ratios and Proportional Relationships 7.RP</u>** **Analyze proportional relationships and use them to solve real-world and mathematical problems.** 7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour or apply a given scale factor to find missing dimensions of similar figures.* 7.RP.2. Recognize and represent proportional relationships between quantities. Make basic inferences or logical predictions from proportional relationships. a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships in real world situations.	

Grade 6	Grade 7	Grade 8
6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations). a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios, and understand equivalencies. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units between given measurement systems (e.g., convert kilometers to miles); manipulate and transform units appropriately when multiplying or dividing quantities.	c. Represent proportional relationships by equations and multiple representations such as tables, graphs, diagrams, sequences, and contextual situations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$. d. Understand the concept of unit rate and show it on a coordinate plane. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate. 7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	

Grade 6	Grade 7	Grade 8
<u>The Number System 6.NS</u> **Apply and extend previous understandings of multiplication and division to divide fractions by fractions.** 6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions (e.g., by using visual fraction models and equations to represent the problem). *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3 (In general (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*	**<u>The Number System 7.NS</u>** **Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** 7.NS.1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. Multiply and divide rational numbers and include other models to the representation. a. Show that a number and its opposite have a sum of 0 (additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* b. Understand addition of rational numbers ($p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	**<u>The Number System 8.NS</u>** **Know that there are numbers that are not rational, and approximate them by rational numbers.** 8.NS.1. Classify real numbers as either rational (the ratio of two integers, a terminating decimal number, or a repeating decimal number) or irrational. 8.NS.2. Order real numbers, using approximations of irrational numbers, locating them on a number line. *For example, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* 8.NS.3. Identify or write the prime factorization of a number using exponents. L

Grade 6	Grade 7	Grade 8
Compute fluently with multi-digit numbers and find common factors and multiples. 6.NS.2. Fluently multiply and divide multi-digit whole numbers using the standard algorithm. Express the remainder as a whole number, decimal, or simplified fraction; explain or justify your choice based on the context of the problem. 6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. Express the remainder as a terminating decimal, or a repeating decimal, or rounded to a designated place value. 6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*	7.NS.2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers and use equivalent representations. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply and name properties of operations used as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. e. Convert between equivalent fractions, decimals, or percents.	

Grade 6	Grade 7	Grade 8
Apply and extend previous understandings of numbers to the system of rational numbers. 6.NS.5 Understand that positive and negative numbers describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explain the meaning of 0 in each situation.	7.NS.3. Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) *For example, use models explanations, number lines, real life situations, describing or illustrating the effect of arithmetic operations on rational numbers (fractions, decimals).*	
6.NS.6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; Recognize that the opposite of the opposite of a number is the number itself [e.g., –(–3) = 3] and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.		

Grade 6	Grade 7	Grade 8
6.NS.7. Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3\ ^{\circ}C > -7\ ^{\circ}C$ to express the fact that $-3\ ^{\circ}C$ is warmer than $-7\ ^{\circ}C$.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write $\|-30\| = 30$ to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.* 6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.		

Grade 6	Grade 7	Grade 8
<u>**Expressions and Equations 6.EE**</u> **Apply and extend previous understandings of arithmetic to algebraic expressions.** 6.EE.1. Write and evaluate numerical expressions involving whole-number exponents *For example multiply by powers of 10 and products of numbers using exponents. ($7^3 = 7•7•7$)* 6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions and formulas. Include formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order with or without parentheses. (Order of Operations)	<u>**Expressions and Equations 7.EE**</u> **Use properties of operations to generate equivalent expressions.** 7.EE.1. Apply properties of operations as strategies to add, subtract, factor, expand and simplify linear expressions with rational coefficients. 7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** 7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*	<u>**Expressions and Equations 8.EE**</u> **Work with radicals and integer exponents.** 8.EE.1. Apply the properties (product, quotient, power, zero, negative exponents, and rational exponents) of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.* 8.EE.2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational. 8.EE.3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.* 8.EE.4. Perform operations with numbers expressed in scientific notation, including problems where both standard notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.

Grade 6	Grade 7	Grade 8
6.EE.3. Apply the properties of operations to generate equivalent expressions. Model (e.g., manipulatives, graph paper) and apply the distributive, commutative, identity, and inverse properties with integers and variables by writing equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x.* 6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*	7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct multi-step equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	**Understand the connections between proportional relationships, lines, and linear equations.** 8.EE.5. Graph linear equations such as $y=mx+b$, interpreting m as the slope or rate of change of the graph and b as the y-intercept or starting value. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* 8.EE.6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Grade 6	Grade 7	Grade 8
Reason about and solve one-variable equations and inequalities. 6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. *For example: does 5 make 3x > 7 true?* 6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. 6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers. 6.EE.8. Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.		**Analyze and solve linear equations and pairs of simultaneous linear equations.** 8.EE.7. Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms. 8.EE.8. Analyze and solve systems of linear equations. a. Show that the solution to a system of two linear equations in two variables is the intersection of the graphs of those equations because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables and estimate solutions by graphing the equations. Simple cases may be done by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Grade 6	Grade 7	Grade 8
Represent and analyze quantitative relationships between dependent and independent variables. 6.EE.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*		

Grade 6	Grade 7	Grade 8
		Functions 8.F **Define, evaluate, and compare functions.** 8.F.1. Understand that a function is a rule that assigns to each input (the domain) exactly one output (the range). The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. *For example, use the vertical line test to determine functions and non-functions.* 8.F.2. Compare properties of two functions function each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* 8.F.3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Grade 6	Grade 7	Grade 8
		Use functions to model relationships between quantities. 8.F.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.F.5. Given a verbal description between two quantities, sketch a graph. Conversely, given a graph, describe a possible real-world example. *For example, graph the position of an accelerating car or tossing a ball in the air.*

Grade 6	Grade 7	Grade 8
<u>Geometry 6.G</u> **Solve real-world and mathematical problems involving area, surface area, and volume.** 6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing or decomposing into other polygons (e.g., rectangles and triangles). Apply these techniques in the context of solving real-world and mathematical problems. 6.G.2. Apply the standard formulas to find volumes of prisms. Use the attributes and properties (including shapes of bases) of prisms to identify, compare or describe three-dimensional figures including prisms and cylinders. 6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; determine the length of a side joining the coordinates of vertices with the same first or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	<u>Geometry 7.G</u> **Draw, construct, and describe geometrical figures and describe the relationships between them.** 7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. 7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes including polygons and circles with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. 7.G.3. Describe the two-dimensional figures, i.e., cross-section, that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	<u>Geometry 8.G</u> **Understand congruence and similarity using physical models, transparencies, or geometry software.** 8.G.1. Through experimentation, verify the properties of rotations, reflections, and translations (transformations) to figures on a coordinate plane). a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. 8.G.2. Demonstrate understanding of congruence by applying a sequence of translations, reflections, and rotations on two-dimensional figures. Given two congruent figures, describe a sequence that exhibits the congruence between them. 8.G.3 .Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Grade 6	Grade 7	Grade 8
6.G.4. Represent three-dimensional figures (e.g., prisms) using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. 6. G.5. Identify, compare or describe attributes and properties of circles (radius, and diameter) L	**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** 7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. 7.G.5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. 7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	8.G.4. Demonstrate understanding of similarity, by applying a sequence of translations, reflections, rotations, and dilations on two-dimensional figures. Describe a sequence that exhibits the similarity between them. 8.G.5. Justify using informal arguments to establish facts about • the angle sum of triangles (sum of the interior angles of a triangle is 180º) • measures of exterior angles of triangles, • angles created when parallel lines are cut be a transversal (e.g., alternate interior angles) and • angle-angle criterion for similarity of triangles. **Understand and apply the Pythagorean Theorem.** 8.G.6. Explain the Pythagorean Theorem and its converse. 8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Grade 6	Grade 7	Grade 8
		8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** 8.G.9. Identify and apply the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
<u>**Statistics and Probability 6.SP**</u> **Develop understanding of statistical variability.** 6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* 6.SP.2 Understand that a set of data has a distribution which can be described by its center (mean, median, or mode), spread (range), and overall shape and can be used to answer a statistical question.	<u>**Statistics and Probability 7.SP**</u> **Use random sampling to draw inferences about a population.** 7.SP.1. Understand that statistics can be used to gain information about a population by examining a reasonably sized sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. 7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*	<u>**Statistics and Probability 8.SP**</u> **Investigate patterns of association in bivariate data.** 8.SP.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. 8.SP.2. Explain why straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Grade 6	Grade 7	Grade 8
6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation (range) describes how its values vary with a single number. **Summarize and describe distributions.** 6.SP.4. Display numerical data in plots on a number line, including dot or line plots, histograms and box (box and whisker) plots. 6.SP.5 Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations (occurrences). b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range), as well as describing any overall pattern and any outliners with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. 6.SP.6 Analyze whether a game is mathematically fair or unfair by explaining the probability of all possible outcomes. L	**Draw informal comparative inferences about two populations.** 7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* 7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	8.SP.3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and y-intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* 8.SP.4. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects and use relative frequencies to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Grade 6	Grade 7	Grade 8
6.SP.7. Solve or identify solutions to problems involving possible combinations (e.g., if ice cream sundaes come in 3 flavors with 2 possible toppings, how many different sundaes can be made using only one flavor of ice cream with one topping?) L	**Investigate chance processes and develop, use, and evaluate probability models.** 7.SP.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. 7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* 7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Design a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*	

Grade 6	Grade 7	Grade 8
	b. Design a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* 7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	

Alaska High School Mathematics Standards

The high school standards specify the mathematics that all students should study in order to be career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by the symbol +.

Math domains for high school are not courses but conceptual categories, which are intended to portray a coherent view of high school mathematics. A student's work with any set of standards crosses a number of traditional course boundaries. For example, the Functions Standards would apply to many courses. It is a district decision how to design course offerings covering the mathematics standards. Districts can use the traditional approach of Algebra I, Geometry, and Algebra II or implement an integrated approach.

Standards with connections to modeling are indicated by *. Modeling is best interpreted in relation to other standards. Making mathematical models is a Standard for Mathematical Practice. If the star symbol appears on the heading for a group of standards, it should be understood to apply to all standards in that group.

Coding for High School Mathematical Domains:

1. Number and Quantity - N
2. Algebra - A
3. Functions – F
4. Modeling - M
5. Geometry - G
6. Statistics and Probability - P

Narrative of Standards - Number and Quantity

Numbers and Number Systems. During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Number and Quantity Standards

The Real Number System	N –RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities*	N –Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

2. Define appropriate quantities for the purpose of descriptive modeling.

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System	N -CN

Perform arithmetic operations with complex numbers.

1. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(1 - \sqrt{3}i)^3 = 8$ *because* $(1 - \sqrt{3}i)$ *has modulus* 2 *and argument* 120°.

6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.

8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.

9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities	N –VM

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*).

2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. (+) Add and subtract vectors.

   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

   c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

5. (+) Multiply a vector by a scalar.

   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

   b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

8. (+) Add, subtract, and multiply matrices of appropriate dimensions.

9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

12. (+) Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Narrative of Standards - Algebra

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Seeing Structure in Expressions	A-SSE

Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.*

   a. Interpret parts of an expression, such as terms, factors, and coefficients.

   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

   a. Factor a quadratic expression to reveal the zeros of the function it defines. *For example,* $x^2 + 4x + 3 = (x + 3)(x + 1)$.

   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. For example, $x^2 + 4x + 3 = (x+2)^2 - 1$.

   c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**

Arithmetic with Polynomials and Rational Expressions A -APR

Perform arithmetic operations on polynomials

1. Add, subtract, and multiply polynomials. Understand that polynomials form a system similar to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.

Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*

5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

7. (+) Add, subtract, multiply, and divide rational expressions. Understand that rational expressions form a system similar to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

Creating Equations*	A –CED

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing cost constraints in various situations.*

4. Solve literal equations or formulas for a variable involving multi-steps. *For example, solve for h when* $A = \frac{1}{2}h(b_1 + b_2)$.

Reasoning with Equations and Inequalities	A –REI

Understand solving equations as a process of reasoning and explain the reasoning

1. Apply properties of mathematics to justify steps in solving equations in one variable.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

4. Solve quadratic equations in one variable.

   a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations

5. Show that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

6. Solve systems of linear equations exactly and approximately, e.g., with graphs or algebraically, focusing on pairs of linear equations in two variables.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$.

8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.

9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Narrative of Standards - Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates. Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Interpreting Functions	F-IF

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities,

   - interpret key features of graphs and tables in terms of the quantities, and
   - sketch graphs showing key features given a verbal description of the relationship.

   Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior. *

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then negative numbers would be an appropriate domain for the function.* *

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

   c. Graph polynomial functions, identifying zeros (using technology) or algebraic methods when suitable factorizations are available, and showing end behavior.

   d. (+) Graph rational functions, identifying zeros and discontinuities (asymptotes/holes) using technology, and algebraic methods when suitable factorizations are available, and showing end behavior.

   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions	**F-BF**

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

4. Find inverse functions.

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *for* $x > 0$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.

b. (+) Verify by composition that one function is the inverse of another.

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models* F –LE

Construct and compare linear, quadratic, and exponential models and solve Problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

   a. Show that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or input-output table of values.
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*

Prove and apply trigonometric identities

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.

9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Narrative of Standard – Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

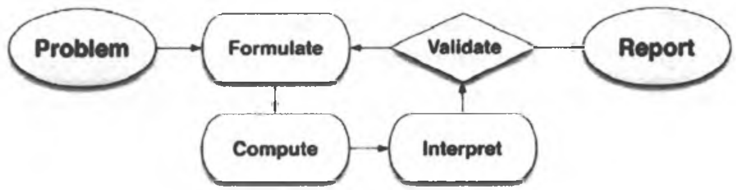
The basic modeling cycle is summarized in the diagram (below). It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).



Narrative of Standard - Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to nonright triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Congruence	G-CO

Experiment with transformations in the plane

1. Demonstrates understanding of key geometrical definitions, including angle, circle, perpendicular line, parallel line, line segment, and transformations in Euclidian geometry. Understand undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, SSS, AAS, and HL) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems

9. Using methods of proof including direct, indirect, and counter examples to prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

10. Using methods of proof including direct, indirect, and counter examples to prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

11. Using methods of proof including direct, indirect, and counter examples to prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, and Trigonometry	**G-SRT**

Understand similarity in terms of similarity transformations

1. Verify experimentally the properties of dilations given by a center and a scale factor:

   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

2. Given two figures, use the definition of similarity in terms of transformations to explain whether or not they are similar.

3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity

4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely.*

5. Apply congruence and similarity properties and prove relationships involving triangles and other geometric figures.

Define trigonometric ratios and solve problems involving right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

7. Explain and use the relationship between the sine and cosine of complementary angles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

Apply trigonometry to general triangles

9. (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.

11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles	**G-C**

Understand and apply theorems about circles

1. Prove that all circles are similar.

2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

4. (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles

5. Use and apply the concepts of arc length and areas of sectors of circles. Determine or derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations **G-GPE**

Translate between the geometric description and the equation for a conic section

1. Determine or derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

2. Determine or derive the equation of a parabola given a focus and directrix.

3. (+) Derive the equations of ellipses and hyperbolas given foci and directrices.

Use coordinates to prove simple geometric theorems algebraically

4. Perform simple coordinate proofs. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*

Geometric Measurement and Dimension	G-GMD

Explain volume formulas and use them to solve problems

1. Explain how to find the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*

Visualize relationships between two-dimensional and three-dimensional objects

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry	G-MG

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*

3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

Narrative of Standard - Statistics and Probability*

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Statistics and Probability Standards*

Interpreting Categorical and Quantitative Data	S-ID

Summarize, represent, and interpret data on a single count or measurement variable

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).

2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Summarize, represent, and interpret data on two categorical and quantitative variables

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

   b. Informally assess the fit of a function by plotting and analyzing residuals.

   c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

8. Compute (using technology) and interpret the correlation coefficient of a linear fit.

9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions	S-IC

Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

6. Evaluate reports based on data.

Conditional Probability and the Rules of Probability	S-CP

Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model

6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model.

7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions	**S-MD**

Calculate expected values and use them to solve problems

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decisions

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*

   b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Glossary for Alaska Mathematics Standards

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team. Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Cardinality/Ordinality. Cardinal numbers, known as the "counting numbers," indicate quantity. Ordinal numbers indicate the order or rank of things in a set (e.g., sixth in line; fourth place).

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See*: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form a or –a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: $72 \div 8 = 9$.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because $3/4 \times 4/3 = 4/3 \times 3/4 = 1$.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or – a/b for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3,...